1. Demonstre as seguintes identidades:

   (a) $\mathcal{F}(f(x-c))(\xi) = e^{-2\pi i c\xi}\hat{f}(\xi)$

   (b) $\mathcal{F}(e^{2\pi i cx}f(x))(\xi) = \hat{f}(\xi - c)$

   (c) $\mathcal{F}(f(ax))(\xi) = \dfrac{1}{a}\hat{f}\left(\dfrac{\xi}{a}\right)$

   (d) $\mathcal{F}(f')(\xi) = 2\pi i \xi \hat{f}(\xi)$

   (e) $\mathcal{F}(xf(x))(\xi) = \dfrac{i}{2\pi}(\hat{f})'(\xi)$

2. Calcule a transformada de Fourier das seguintes funções:

   (a) $f(x) = \dfrac{1}{x^2 + a^2}$

   (b) $f(x) = e^{-a|x|}$