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Late points for random walks, and the fluctuations of cover times

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Consider a random walk over a finite state space such as a graph or digraph. The cover time is the time it takes to visit all states, and finding its asymptotic distribution has been an important problem in the theory of finite Markov chains. Even the concrete the case of the $3d$ torus was open for nearly 20 years until Belius's recent proof of Gumbel fluctuations.

In this talk we will present a large class of Markov chains for which we can solve the problem of fluctuations of cover times. In fact, we obtain much stronger results that describe the distribution of the uncovered set, the set of points not yet visited by the random walker. We use a rigorous mean-field approach to prove that the uncovered set at "late times" is a time-evolving Poisson cloud, and that points leave that set at specific rates, independently of one another. In particular, the law of the cover time follows from the fact that the number of uncovered points is Poisson.

Our approach gives new results even for the $3d$ torus; for instance, we show that the last k points visited are essentially uniformly distributed. We also obtain results for large-girth expander graphs; random graphs with given degrees; patterns of length n in iid coin tossing; and other examples where random walk is rapidly mixing and locally transient (in a sense that we will make precise in the talk).

(This is joint work with Alan Prata from IMPA. Some of the results are in his PhD thesis, which should be in IMPA's website).