On infinite energy solutions of Schrödinger-type Equations with a nonlocal term

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Joint work with Ademir Pastor (UNICAMP)

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Schrödinger-type Equations with a nonlocal term

Initial value problem (IVP) associated with Schrödinger-type equations of the form

$$\begin{cases} i\partial_t u + Lu = \chi |u|^\rho u + bE(|u|^\rho)u, \\ u(x,0) = u_0(x). \end{cases} \tag{1}$$

- $(x,t) \in \mathbb{R}^n \times \mathbb{R}, \ n \geq 1,$
- $u = u(x,t)$ is a complex-valued function,
- χ and b are real constants, ρ is a positive real number
- L and E are linear operators.

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Some examples

- Schrödinger equation

$$i\partial_t u + \Delta u = \chi |u|^\rho u. \tag{2}$$

- Davey-Stewartson system $(n \geq 2,\ m > 0)$

$$\begin{cases} i\partial_t u + \delta \partial_{x_1}^2 u + \sum_{j=2}^n \partial_{x_j}^2 u = \chi |u|^\rho u + b u \partial_{x_1} \varphi, \\ \partial_{x_1}^2 \varphi + m \partial_{x_2}^2 \varphi + \sum_{j=3}^n \partial_{x_j}^2 \varphi = \partial_{x_1}(|u|^\rho), \\ u(x,0) = u_0(x). \end{cases} \tag{3}$$

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- To $m > 0$, system (3) becomes

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where

$$\widehat{E(f)}(\xi) = \frac{\xi_1^2}{\xi_1^2 + m\xi_2^2 + \sum_{j=3}^n \xi_j^2}\hat{f}(\xi) = p(\xi)\hat{f}(\xi).$$

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Motivation

In [B]

$$i\partial_t u + \delta \partial_{x_1}^2 u + \sum_{j=2}^{n} \partial_{x_j}^2 u = \chi |u|^\rho u + buE(|u|^\rho)$$

- Lorentz spaces:
  $L^{p\infty}(\mathbb{R}^n) = \{f; \|f\|_{L^{p\infty}(\mathbb{R}^n)} := \sup_{\lambda>0} \lambda \alpha(\lambda, f)^{1/p} < \infty\}$
  $\alpha(\lambda, f) = \mu(\{x \in \mathbb{R}^n; |f(x)| > \lambda\})$,
- Global in time solutions ($\Rightarrow$ self-similar solutions)

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$$i\partial_t u + \Delta u = \chi |u|^\rho u.$$

- $E_\alpha = \{u; \|u\|_\alpha = \sup_{-\infty<t<+\infty} |t|^{\alpha/2} \|u(t)\|_{L^{(\rho+2,\infty)}} < \infty\}$, where $\alpha = \frac{2}{\rho} - \frac{n}{\rho+2}$.
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$$i\partial_t u + \delta \partial_{x_1}^2 u + \sum_{j=2}^{n} \partial_{x_j}^2 u = \chi |u|^\rho u + buE(|u|^\rho)$$

Extends the results of [SFR] to the D-S system.

$L = \delta \partial_{x_1}^2 + \sum_{j=2}^{n} \partial_{x_j}^2 \to L$

$\widehat{E(f)}(\xi) = \frac{\xi_1^2}{\xi_1^2 + m\xi_2^2 + \sum_{j=3}^{n} \xi_j^2} \hat{f}(\xi) = p(\xi)\hat{f}(\xi) \to E$

To give sufficient conditions on the operators L and E that allow to extend the results to the IVP (1) .

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Conditions on L and E

L is a pseudo-differential operator defined via its Fourier transform by

$$\widehat{Lu}(\xi) = q(\xi)\widehat{u}(\xi), \tag{4}$$

(H1) the function q is real and homogeneous of degree d, that is,

$$q(\lambda\xi) = \lambda^d q(\xi), \qquad \lambda > 0.$$

(H2) The function $G(x) = \int_{\mathbb{R}^n} e^{i(x\xi+q(\xi))}d\xi$ belongs to $L^\infty(\mathbb{R}^n)$.
(H3) E is bounded from $L^{(p,\infty)}(\mathbb{R}^n)$ to itself, for all p satisfying $1 < p < \infty$.

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- Self-similar solutions + conditions,
- Our main results,
- Applications.

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Self-similar solutions

If $u(x,t)$ is a solution of (1) so is $u_\lambda(x,t) = \lambda^{d/\rho}u(\lambda x, \lambda^d t)$, for any $\lambda > 0$.
important!**(H1)**-$q(\lambda\xi) = \lambda^d q(\xi)$

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Conditions to have Self-similar solutions

Supposing

- $\exists!$ of solutions to the IVP problem (1)
- u a self-similar solution

We must have

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i.e., $u_0(x) = \lambda^{d/\rho} u_0(\lambda x)$ (u_0 is homogeneous).

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Main Results

Integral equivalent formulation to the IVP (1)

$$u(t) = U(t)u_0 + i\int_0^t U(t-s)(\chi|u|^\rho u + buE(|u|^\rho))(s)ds, \qquad (5)$$

where $U(t)u_0$ is the solution of the linear problem

$$\begin{cases} i\partial_t u + Lu = 0, \\ u(x,0) = u_0(x) \end{cases} \qquad (x,t) \in \mathbb{R}^n \times \mathbb{R}, \qquad (6)$$

that is,

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Main Results-Global Existence

Theorem (Global Existence)

- $1 < \rho < \infty$ *and* $\frac{\rho+2}{\rho+1} < \frac{n\rho}{d} < \rho + 2$,
- ϕ *is a distribution satisfying* $\|U(t)\phi\|_\alpha \leq \epsilon$, *where* $0 < \epsilon << 1$.

Then

- *The integral equation (5) has a unique solution* $u \in E_\alpha$ *satisfying* $\|u\|_\alpha \leq 2\epsilon$, *where* $E_\alpha = \{u; \|u\|_\alpha = \sup_{-\infty<t<+\infty} |t|^{\alpha/d}\|u(t)\|_{L(\rho+2,\infty)} < \infty\}$,

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$$i\partial_t u + \Delta u = \chi |u|^\rho u.$$

Global solutions: $0 < \rho < \infty$ and $\frac{\rho+2}{\rho+1} < \frac{n\rho}{d} < \rho + 2.$

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$$u(t) = U(t)\phi + i\int_0^t U(t-s)(\chi|u|^\rho u + buE(|u|^\rho))(s)ds,$$

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Lemma (A)

- $1 < p < 2$,
- p' *such that* $\frac{1}{p} + \frac{1}{p'} = 1$.

Then there exists a constant $C = C(n, p) > 0$ *such that*

- $\|U(t)\phi\|_{L^{(p',\infty)}} \leq C|t|^{-\frac{n}{d}(\frac{2}{p}-1)}\|\phi\|_{L^{(p,\infty)}}$,
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$$B(u) = i \int_0^t U(t-s)(\chi |u|^\rho u + buE(|u|^\rho))(s)ds.$$

Lemma (B)

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Then there exists a positive constant K_α *such that*

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- Lemma (A) + **(H3)** E is bounded from $L^{(p,\infty)}(\mathbb{R}^n)$ to itself, for all p satisfying $1 < p < \infty$.

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- Lemma (A) + **(H3)** E is bounded from $L^{(p,\infty)}(\mathbb{R}^n)$ to itself, for all p satisfying $1 < p < \infty$.

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$$B(u) = i\int_0^t U(t-s)(\chi|u|^\rho u + buE(|u|^\rho))(s)ds.$$

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- $1 < \rho < \infty$ *and* $\frac{\rho+2}{\rho+1} < \frac{n\rho}{d} < \rho + 2$

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Lemma (B) + hypothesis $\|U(t)\phi\|_\alpha \leq \epsilon$ implies

- $\Phi : \overline{B}(0, 2\epsilon) \to \overline{B}(0, 2\epsilon)$ well defined.
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We need to ask ϕ homogeneous

- $\phi(\lambda x) = \lambda^{-\frac{d}{\rho}}\phi(x), \ \forall\ \lambda > 0.$

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$$t^{\frac{\alpha}{d}} \|U(t)\phi\|_{L^{(\rho+2,\infty)}} = \|U(1)\phi\|_{L^{(\rho+2,\infty)}},$$

Corollary (Self-similar solutions)

- $1 < \rho < \infty$ *and* $\frac{\rho+2}{\rho+1} < \frac{n\rho}{d} < \rho + 2$,
- ϕ *is homogeneous of degree* $-d/\rho$,
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Then

- *the solution* u *obtained in Theorem Global Existence is self-similar.*

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Other Results-Scattering

Theorem (Scattering)

$$\|u(t) - U(t)u_\pm\|_{L^{(\rho+2,\infty)}} \leq C|t|^{-\frac{\alpha}{d}}\|u\|_\alpha^{\rho+1}, \quad t \neq 0.$$

$$u \longrightarrow v(t) = U(t)\phi + B(v)$$

$$U(t)u_\pm \longrightarrow \begin{cases} i\partial_t v + Lv = 0, \\ v(0) = u_\pm, \end{cases}$$

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$$\lim_{|t|\to\infty} |t|^{\frac{\alpha}{d}+\delta} \|\phi - \tilde{\phi}\|_{L^{(\rho+2,\infty)}} = 0$$

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Applications

Standard NLS Equation ($n \geq 1$)

$$i\partial_t u + \Delta u = \chi u|u|^\rho, \quad (x,t) \in \mathbb{R}^n \times \mathbb{R},\ n \geq 1,$$

[SFR]

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(H1) the function $q(x) = {x_1}^2 - {x_2}^2$ homogeneous of degree 2,
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$$i\partial_t u + \delta\partial_{x_1}^2 u + \delta\partial_{x_2}^2 u = \chi|u|^\rho u + buE(|u|^\rho), n = 2,$$

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The Shrira system $(n = 3)$

$$\begin{cases} i\partial_t u + \frac{\omega_{kk}}{2}\partial_x^2 u + \frac{\omega_{\ell\ell}}{2}\partial_y^2 u + \frac{\omega_{nn}}{2}\partial_z^2 u + \omega_{nk}\partial_{xz}^2 u = -uQ, \\ \\ \partial_x^2 Q + \partial_y^2 Q = \nu \partial_y^2 |u|^\rho, \end{cases}$$

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