Questão 2

$$\mathcal{L}\{y''+y\} = \mathcal{L}\{u_{\frac{\pi}{2}}(t) + 3\,\delta(t-\tfrac{3\pi}{2})\}$$

$$0{,}1 \quad \Big[\; \underbrace{\underbrace{\mathcal{L}\{y''\}}_{s^2\mathcal{L}\{y\}-sy(0)-y'(0)}}_{0{,}2} + \mathcal{L}\{y\} = \underbrace{\underbrace{\mathcal{L}\{u_{\frac{\pi}{2}}(t)\}}_{\frac{e^{-\frac{\pi}{2}s}}{s}}}_{0{,}2} + 3\,\underbrace{\underbrace{\mathcal{L}\{\delta(t-\tfrac{3\pi}{2})\}}_{e^{-\frac{3\pi}{2}s}}}_{0{,}2}$$

$$y(0) = y'(0) = 0$$

$$0{,}1 \quad \Big[\; \therefore \quad (s^2+1)\mathcal{L}\{y\} = \frac{e^{-\frac{\pi}{2}s}}{s} + 3e^{-\frac{3\pi}{2}s}$$

$$\mathcal{L}\{y\} = \frac{e^{-\frac{\pi}{2}s}}{s(s^2+1)} + 3\,\frac{e^{-\frac{3\pi}{2}s}}{s^2+1}$$

$$\left.\begin{aligned} &\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \operatorname{sen} t \\ &\left(\mathcal{L}\{\operatorname{sen} t\} = \frac{1}{s^2+1}\right)\end{aligned}\right]\; 0{,}2$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-\frac{3\pi}{2}s}}{s^2+1}\right\} = u_{\frac{3\pi}{2}}(t)\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}(t-\tfrac{3\pi}{2}) \quad \Big]\; 0{,}2$$

$$= u_{\frac{3\pi}{2}}(t)\,\operatorname{sen}(t-\tfrac{3\pi}{2}) \quad \Big]\; 0{,}1$$

$$0{,}1 \quad \Big[\; \frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$= \frac{A(s^2+1) + s(Bs+C)}{s(s^2+1)}$$

$$1 = (A+B)s^2 + Cs + A \quad \therefore \quad \left\{\begin{aligned} A &= 1 \\ C &= 0 \\ B &= -1 \end{aligned}\right. \quad \Big]\; 0{,}1$$

$$\left.\begin{aligned} \therefore \quad \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} &= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1\,s}{s^2+1}\right\} \\ &= 1 - \operatorname{sen} t \end{aligned}\right]\; 0{,}2$$

cos t

corrigir

$$\mathcal{L}^{-1}\left\{\frac{e^{-\frac{\pi}{2}s}}{s(s^2+1)}\right\} = u_{\frac{\pi}{2}}(t)\,\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\}(t-\tfrac{\pi}{2})$$

$$= u_{\frac{\pi}{2}}(t)\cdot\left[1 - \overset{\cos}{\cancel{\operatorname{sen}}}(t-\tfrac{\pi}{2})\right] \quad \Big]\; 0{,}2$$

Resposta:

$$\boxed{y = u_{\frac{\pi}{2}}(t)\cdot\left[1-\overset{\cos}{\cancel{\operatorname{sen}}}(t-\tfrac{\pi}{2})\right] + 3\,u_{\frac{3\pi}{2}}(t)\,\operatorname{sen}(t-\tfrac{3\pi}{2})} \quad \Big]\; 0{,}1$$