

Cálculo da integral $\int_{E(1)} \frac{|y|^2}{s^2} dy ds = 4$,

$$E(1) = \{(y, s) \in \mathbb{R}^{n+1}; |y| \leq \rho(s), -\frac{1}{4\pi} \leq s \leq 0\}, \quad \rho(s) := (2ns \ln(-4\pi s))^{1/2}$$

$$\begin{aligned} & \int_{E(1)} \frac{|y|^2}{s^2} dy ds = \int_{-\frac{1}{4\pi}}^0 \int_{|y| \leq \rho(s)} \frac{|y|^2}{s^2} dy ds \\ &= \int_{-\frac{1}{4\pi}}^0 \omega_n \int_0^{\rho(s)} \frac{r^2}{s^2} r^{n-1} dr ds = \omega_n \int_{-\frac{1}{4\pi}}^0 \frac{1}{s^2} \frac{\rho(s)^{n+1}}{n+2} ds \\ &= \frac{\omega_n}{n+2} \int_{-\frac{1}{4\pi}}^0 \frac{1}{s^2} (2ns \ln(-4\pi s))^{(n+2)/2} ds \quad (\text{mudança de variável: } -\tau = \ln(-4\pi s)) \\ &= \frac{\omega_n}{n+2} (4\pi)^{-n/2} (2n)^{(n+2)/2} \int_0^\infty \tau^{\frac{n+2}{2}} e^{-\frac{n}{2}\tau} d\tau \quad (\text{mudança de variável: } s = \frac{n}{2}\tau) \\ &= \frac{\omega_n}{n+2} (4\pi)^{-n/2} (2n)^{(n+2)/2} \left(\frac{2}{n}\right)^{(n+4)/2} \int_0^\infty s^{\frac{n}{2}+1} e^{-s} ds \\ &= \frac{\omega_n}{n+2} (4\pi)^{-n/2} (2n)^{(n+2)/2} \left(\frac{2}{n}\right)^{(n+4)/2} \Gamma\left(\frac{n}{2} + 2\right) \quad (\Gamma \text{ é a função Gama}) \\ &= \frac{\omega_n}{n+2} (4\pi)^{-n/2} (2n)^{(n+2)/2} \left(\frac{2}{n}\right)^{(n+4)/2} \left(\frac{n}{2} + 2\right) \Gamma\left(\left(\frac{n}{2} + 1\right)\right) \\ & \quad (\text{usando que } \Gamma(x+1) = x\Gamma(x)) \\ &= 4 \quad (\text{usando a fórmula } \omega_n = n\pi^{n/2}/\Gamma(\frac{n}{2} + 1); \text{ v. Apêndice A.2 do [Evans]}). \end{aligned}$$