Nome: ______________________________ RA: ____________________

Métodos Matemáticos I (F520/MS550) - Teste 6

16 de junho de 2010

1. Seja P_n o n-ésimo polinômio de Legendre ($n = 0, 1, 2, 3 \ldots$). Calcule:

   (a) $P_n'(1)$.

   (b) $P_n(0)$.

   (c) $\int_0^1 P_n(x)dx$.

(a) Da eq. diferencial de Legendre:

$$(1-x^2)P_n'' - 2x\,P_n' + n(n+1)P_n = 0$$

Avaliando em $x=1$, temos:

$$0 - 2\cdot 1\cdot P_n'(1) + n(n+1)\underbrace{P_n(1)}_{1} = 0$$

$$\Rightarrow \quad P_n'(1) = \frac{n(n+1)}{2}.$$

(b) Da função geratriz,

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)\,t^n,$$

série binomial

temos
$$\sum_{n=0}^{\infty} P_n(0)\,t^n = (1+t^2)^{-1/2} =$$

$$= 1 + \left(-\frac{1}{2}\right)t^2 + \frac{1}{2!}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)t^4 + \cdots +$$

$$+ \frac{1}{m!}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdots\left(-\frac{1}{2}-m+1\right)t^{2m} + \cdots$$

$$= 1 - \frac{1}{2}t^2 + \frac{3}{8}t^4 + \cdots + (-1)^m\frac{1\cdot 3\cdots(2m-1)}{m!\,2^m}t^{2m} + \cdots$$

Logo
$$P_n(0) = \begin{cases} 0, & \text{se } n \text{ é ímpar} \\ (-1)^m \dfrac{1\cdot 3\cdot 5\cdots(2m-1)}{2^m\,m!}, & \text{se } n \text{ é par} \end{cases}$$

$t>0$

(c)
$$\sum_{n=0}^{\infty}\left(\int_0^1 P_n(x)\,dx\right)t^n = \int_0^1 \frac{dx}{\sqrt{1-2xt+t^2}} = \frac{1}{-2t}\left[\sqrt{1-2xt+t^2}\right]_0^1 = \frac{1}{t}\left(t-1+\sqrt{1+t^2}\right)$$

$$\Rightarrow \sum_{n=0}^{\infty}\left(\int_0^1 P_n(x)\,dx\right)t^{n+1} = t - \cancel{1} + \left\{\cancel{1} + \frac{1}{2}t^2 + \frac{1}{2!}\,\frac{1}{2}\left(-\frac{1}{2}\right)t^4 + \cdots + \frac{1}{k!}\,\frac{1}{2}\left(-\frac{1}{2}\right)\cdots\left(\frac{1}{2}-k+1\right)t^{2k}\right\}$$

$$\Rightarrow \int_0^1 P_{2m}(x)\,dx = \begin{cases} 1 & \text{se } m=0 \\ 0 & \text{se } m>0 \end{cases} \quad \text{e} \quad \int_0^1 P_{2k+1}(x)\,dx = \frac{(-1)^k}{2^{k+1}}\,\frac{1\cdot 3\cdots(2k-1)}{(k+1)!}$$

$\left(= \frac{1}{2} \text{ se } k=0\right)$