9.2.23. Let $y = x^2 + 1$. Calculate dy/dx and, from the inverse function, dx/dy. Verify $dx/dy = (dy/dx)^{-1}$.

9.2.24. Let $y = x^{3/2}$ for $x > 0$. By using the inverse function verify the formula $dx/dy = (dy/dx)^{-1}$.

9.3.1. Find the antiderivatives of the following functions:

a) $y' = 6x^2$, b) $y' = 8x - 7$, c) $u' = at + b$ (a, b are constants),

d) $\dfrac{dy}{dx} = 5x^3$, e) $\dfrac{dW}{dt} = 2t - 8$, f) $\dfrac{dU}{dx} = U_0 + \cos x$,

g) $y' = \frac{1}{3}\cos t$, h) $U' = \cos 2x$, i) $K' = 1/u^2$.

9.3.2. Find the antiderivatives of the following functions:

a) $y' = 3x^4 - x^2$, b) $y' = 1 - x^{2/3}$,

c) $\dfrac{dy}{dx} = ax^3 - bx^5$, d) $\dfrac{ds}{dt} = a\sqrt{t}$,

e) $\dfrac{du}{ds} = \dfrac{k}{s^3}$, f) $\dfrac{dQ}{d\alpha} = a\cos\alpha + b\sin\alpha$,

g) $\dfrac{dy}{dt} = \dfrac{pt^2 + q}{t^2}$, h) $\dfrac{dP}{dv} = av^2 + bv + c + \dfrac{d}{v^2}$.

9.5.1. Find an approximate value for the integral $\int_1^2 \frac{4}{x}\,dx$ by dot counting. (Use 5 equidistant intervals.)

9.5.2. Plot a graph of the function $y = 5 + 2x - \frac{1}{2}x^2$ over the interval from $x = 0$ to $x = 3$. Find

$$\int_0^3 y\,dx$$

a) approximately by the method of dot counting, b) exactly by integration.

9.5.3. Evaluate the definite integrals

a) $\displaystyle\int_1^2 \frac{1}{r^2}\,dr$, b) $\displaystyle\int_{-1}^1 (5 - w)\,dw$, c) $\displaystyle\int_0^t (at^2 + bt + c)\,dt$,

d) $\displaystyle\int_1^3 t\,dt$, e) $\displaystyle\int_{-1}^1 dx$, f) $\displaystyle\int_{-\pi/2}^{\pi/2} \cos t\,dt$.

9.5.4. Evaluate

a) $\int_0^2 (x^2+1)\,dx$, b) $\int_1^2 \frac{1}{r^2}\,dr$, c) $\int_{-\beta}^{\beta} \sin\alpha\,d\alpha$,

d) $\int_p^q x^2 dx$, e) $\int_a^{a+1} \frac{du}{u^2}$, f) $\int_0^s t^{2/3} dt$.

9.5.5. Find the following indefinite integrals:

a) $\int x^{-6} dx$, b) $\int t^{-1/3} dt$, c) $\int (u+2u^2+3u^3)\,du$,

d) $\int s^{0.1} ds$, e) $\int (A\sin\theta + B\cos\theta)\,d\theta$, f) $\int \frac{dQ}{\sqrt{8Q^3}}$.

9.5.6. Find

a) $\int_0^{10} \frac{du}{(u+2)^2}$, b) $\int_{-p}^{p} A\cdot dr$, c) $\int_1^{3/2} t\,dt$, d) $\int_{-1}^{1} dx$.

9.5.7. Applying the fundamental theorem of integral calculus find the following derivatives:

a) $\frac{d}{dx}\int_0^x \frac{1}{t+2}\,dt$, b) $\frac{d}{dx}\int_1^x \frac{au-1}{u+1}\,du$,

c) $\frac{d}{dt}\int_2^t \frac{u-3}{\sin u}\,du$, d) $\frac{d}{ds}\int_0^s \sqrt{t^3+1}\,dt$.

9.5.8. If a helical spring is extended moderately, Hooke's law is valid. It states that the amount of extension, s, is proportional to the extending force, F, that is, $F = ks$ ($k>0$, constant). Let s be measured in meters and F in Newtons. Then the energy (work) required for the extension is measured in Joules. Find the energy, W, to extend the spring from $s=0$ to $s=s_0$. (Hint: Study Example 9.4.4.)

9.6.1. Find the second derivatives of the following functions:

a) $y=1-x^3$, b) $u=2z^5-3z^3$, c) $W=3/t$, d) $p=2\sqrt{s}$.

9.6.2. Find

a) $\frac{d^2}{dx^2}(ax^3+bx+c)$, b) $\frac{d^2}{dt^2}t^{2/3}$, c) $\frac{d^2}{du^2}\sqrt{u+2}$,

d) $\frac{d^2}{d\alpha^2}\cos(2\alpha+3)$, e) $\frac{d^2}{d\theta^2}(\cos\theta-\sin\theta)$, f) $\frac{d^2}{dr^2}(r-1)^{1/3}$.

9.8.1. Find the mean values of the following functions:

a) $y=\frac{1}{2}x+3$ for $0\leqq x\leqq 6$,
b) $s=at^2$ for $2\leqq t\leqq 3$,
c) $F=\cos\alpha$ for $-\frac{\pi}{2}\leqq\alpha\leqq\frac{\pi}{2}$.

9.8.2. Find the mean values of the following functions:

a) $y=4-\frac{1}{3}x$ for $-2\leqq x\leqq 2$,
b) $U=5+\frac{1}{4}w-\frac{1}{2}w^2$ for $0\leqq w\leqq 3$,
c) $K=r^{-2}$ for $1\leqq r\leqq 3$.

9.9.1. The surface area of a spherical cell is $S=4\pi r^2$ and the volume $V=\frac{4}{3}\pi r^3$. How are S and V affected by a small increase δr of r?

9.9.2. When a muscle contracts against a force F (e.g. a weight), the speed v of shortening decreases with increasing force. A. V. Hill discovered the following equation in 1938:

$$(F+a)(v+b)=c$$

with suitable positive constants a, b, c. Express v in terms of F. How is v affected by a small change δF of F? (For Hill's law see Abbott and Brady, 1964, p. 349).

9.9.3. By inspection of Figure 9.28 prove that for $\delta x>0$

$$(\text{Min}\, f'(x))\cdot\delta x\leqq\delta y\leqq(\text{Max}\, f'(x))\cdot\delta x\,.$$

The minimum and maximum of $f'(x)$ are taken in the interval from x to $x+\delta x$.

*9.10.5. Integrate by substitution

a) $\int\cos\omega x\,dx$, b) $\int(1-8u)^3\,du$, c) $\int\sqrt{p+qs}\,ds$ $(q\neq 0)$,

d) $\int_0^2(4t+3)^4\,dt$, e) $\int_0^{\pi/2}\cos 3\theta\,d\theta$, f) $\int_1^3\frac{dx}{(5x-1)^2}$.

*9.10.6. Integrate by substitution

a) $\int\left(\frac{x}{3}-1\right)^5 dx$, b) $\int(a+bv)^{1/3}\,dv$, c) $\int 1/(3-u)^{1/2}du$,

d) $\int_3^5\frac{dw}{(w-2)^{1/2}}$, e) $\int_0^3\frac{ds}{(4-s)^3}$, f) $\int_{-\pi/2}^{\pi/2}\sin 2\phi\,d\phi$.

*9.10.7. Integrate by parts

a) $\int x\sin x\,dx$, b) $\int t\cos\omega t\,dt$, c) $\int u(u+1)^{1/2}\,du$.

*9.10.8. Integrate by parts

a) $\int_0^\pi x\cos 2x\,dx$, b) $\int_0^{\pi/2}x^2\cos x\,dx$, c) $\int_0^{1/2}t(1-t)^{-1/2}\,dt$.