

A COURSE ON MATHS TEXTBOOK ANALYSIS IN THE TEACHERS TRAINING CURRICULUM: THE EXPERIENCE OF UNICAMP

Henrique Sá Earp¹

Rúbia Barcelos Amaral-Schio²

Abstract

We present the experience of the undergraduate course on Mathematics textbook analysis in the teachers training programme at Unicamp (Brazil) from 2014 to 2016, based on qualitative analysis. Our findings corroborate a number of novel practices both in the lecturing process and in theoretical methodology. Our contribution spans over both intrinsic and comparative analyses of textbooks, the acquisition of good practices from textbooks of various countries, the production of textbook material to a high standard of content and layout and the possibilities of reflexive feedback from students into the course structure itself.

Introduction

The undergraduate teachers training programme (*Licenciatura*) at the University of Campinas (Unicamp) has recently incorporated the one-semester (60h) course on *Analysis of Mathematics textbooks and teaching materials* as a mandatory curriculum requirement. The course had existed before as an optional subject and it has attracted substantial interest from students since its total restructuring, by the first-named author, in 2014. Its objective is to prepare the students, as future teachers, to systematically evaluate textbooks from the perspectives of individual quality, relative quality and consonance with international standards. To that end, a number of novel practices have been introduced, and the aim of this paper is to explain its innovative methodology, based on qualitative analysis.

For our purposes, the term *textbook* designates books as such but it also encompasses similar printed teaching support materials, such as handout sets, examples sheets and so on. The critical examination of the textbooks addresses content, structure, language, layout, examples and exercises, both in variety and in quality.

1- The official system of textbook evaluation in Brazil

The use of standardised textbooks has been ubiquitous in Brazilian classrooms for decades. Following the international norm, the textbook is the teacher's most adopted resource (Valverde, 2002). The primary role of the textbook is to support the teacher, who is free to use it in their own way, integrating it, for example, with other media such as the computer as a tool, as well as online content, video, concrete materials and further bibliography. In Brazil, the assessment and distribution of textbooks are performed by a federal programme under the Ministry of Education.

1.1- The National Textbook Programme (PNLD)

Mazzi and Amaral (2017) produced a historical account of Mathematics textbooks in Brazil, tracing it back to the first known examples and studying the evolution of relevant public policy. While a number of official initiatives at local and national level have been set up over time to

1 Institute of Mathematics, Statistics and Scientific Computing (IMECC) - Unicamp.

2 São Paulo State University (Unesp); collaborator in the Education graduate program at Unicamp.

ensure minimal quality standards, year 1985 marks the consolidation of the current assessment framework, with the establishment of the Brazilian Federal Government's National Textbook Program (PNLD)³. The Programme schedule is organised in three consecutive 3-year cycles, each dedicated to one of the three segments of schooling (Elementary, Middle or High school). Every year, independent authors may submit their collections for the current segment to be certified by the PNLD committee at the Brazilian Ministry of Education (MEC). To do so they must pair up with a publisher, who applies on their behalf for evaluation.

Approved collections are announced by MEC in the *PNLD Textbook Guide*, which is sent to every state school (though not to private schools) in the country and also made publicly available on the Ministry's webpage. Each school adopts independently the collection best suited to their pedagogical project at each level of schooling (Brazil, 2012). Finally, collections are purchased from publishers and sent to every public school pupil. It should be noted that the analysis takes place every three years for each cycle. "Thus, each year, MEC acquires and distributes books for all students in a segment, among Elementary school, Middle school or High school" (Brazil, 2012, p.2, translated by the authors). Distributed books are expected to be retained and returned for use by other students in subsequent years.

It stands to reason that the teacher has an important role in this process, and yet most teachers' training programs in Brazil offer little systematic criteria, if any at all, for that crucial choice. The object of this paper can be seen in this context as a relevant set of skills found severely lacking in the current Brazilian undergraduate teacher training degrees.

1.2- PNLD criteria and procedures

Every PNLD public call establishes a set of criteria for textbook evaluation, with rather explicit sufficient requirements for approval. This includes a common core, applicable to all school subjects, addressing for example universal ethical principles, no reference to religion or merchandising, the presence of some interdisciplinary perspective, actuality of concepts and compliance with a number of laws, regulations and manuals. On the other hand, subject-specific criteria for Mathematics textbooks consist of certain curricular guidelines and a list of cognitive skills to be developed, such as logical thinking, rational argumentation, mathematical modelling and so on. Finally, some few criteria for direct rejection are provided, such as conceptual errors, methodological inconsistencies, and failure to cover all broad subareas of Mathematics, as prescribed by the curricular guidelines (Brazil, 2014; Borba & Selva, 2013).

For most of its existence, the actual PNLD assessment work has been performed at an accredited University, by teams comprising academic specialists in Mathematical Education (from different Brazilian institutions), as well as teachers of the three levels of schooling and other Education professionals. Submitted collections are analysed in a double-blind review process. The referee reports are then examined by the Programme's steering committee, which issues a pass-fail verdict.

2- Textbook analysis in Brazilian teacher training (*Licenciatura* in Mathematics)

In Brazil, teacher training is an independent subject-specific undergraduate degree called *Licenciatura*, which typically lasts 8 to 10 semesters. For the purposes of this article, that term will refer specifically to the *Licenciatura* course in Mathematics.

3 *Programa Nacional do Livro Didático*, in Portuguese.

2.1- Current literature

Rosa, Ribas & Barazzutti (2012) report on the lecturing of a *Licenciatura* course dedicated in part to the evaluation of high school textbooks in Mathematics. Their points of analysis included the tasks proposed in a given book, the methodological approach of the content and its adequacy to the proposed age group. They follow an analysis script somewhat adapted from the PNLD criteria, in which they examine in broad terms:

- The physical characteristics of the textbook, eg. cataloguing data, number of pages etc.
- The diversity in subareas of mathematics, such as numbers and operations, functions, algebraic equations, Euclidean geometry, analytic geometry, statistics and probability etc. Are all fields addressed? Which ones get more and less emphasis? Is this emphasis consistent with the grade for which the book is intended?
- Syllabus and connections between topics. Is content selection appropriate? Are there explicit interconnections between the contents of different chapters?

In addition, a sample of two specific contents is selected by the course students for detailed analysis, according to the previous three axes as well as:

- The quality of content, how it is introduced and developed. Are there links to previous knowledge by the students, and in what form? What types of exercises are offered, which proportion address repetition and memorisation, as opposed to more elaborate problem situations? Do they allow students to test different strategies? Do either contents or exercises have inconsistencies? Is there incentive for student-student or student-teacher interaction in proposed activities? Is there any employment of other teaching resources? Is there any kind of contextualization with social practices and/or other fields of knowledge?

Finally, the authors stress the importance of analysing a textbook before using it in class, since good material can assist in the construction of lesson plans and their time-management, as well as suggest alternative classroom activities. Conversely, they warn that the teaching-learning process may be hindered when the textbook is in disaccord with the objectives set by the teacher, in which case it can drive the teaching proposal astray or limit the exploration breadth around a given concept.

Salla (2012) argues that some undoubtedly relevant aspects such as conceptual errors, biases, methodological inconsistencies, problems in layout and overall lack of standards are already evaluated from the outset by the PNLD process. She suggests that teachers should add a further layer of analysis in terms of:

- how the textbook relates to knowledge that students bring from outside school;
- whether possibilities of discussing alternative resolution strategies are presented;
- if the contents are motivated from a contextual problem, as opposed to an emphasis on algorithmic procedures;
- whether activities encourage experimentation, valuing pupils' individual strategies and enabling logical and autonomous thinking;
- finally, whether there is a balanced presence of the various subareas of Mathematics, while observing that the PNLD Textbook Guide entry for each collection already contains a graph representing their respective proportions.

Lima et al. (2001) is arguably the first systematic effort by a team comprised essentially of mathematicians to engage with school textbook quality. In some sense, it is the only Brazilian reference to offer a category-based grid with definite *a priori* standards stated in objective terms. It structures the intrinsic analysis of a textbook around six main axes. The first one, Concepts, is by far the most present in their actual analyses and it unfolds into painstaking detail, as per the following scheme:

| | | |
|---|---|------------------------------------|
| * Concepts | Mistakes | Typos and misprints |
| | | Faulty reasoning |
| | | Incorrect or incomplete definition |
| | | Poorly stated or vague concepts |
| | Excessive formalism | |
| | Inadequate language | |
| | Imprecision or omission | |
| | Obscurity (ambiguity or self-contradiction) | |
| | Confusion of concepts | |
| | Objectivity (balance between topics) | |
| Connections between topics | | |
| * Manipulation | | |
| * Application | | |
| * Pedagogical qualities | | |
| * Adequacy to contemporary social reality | | |
| * Constructive role and fairness of the assessment process itself | | |

Source: Lima et al. (2001)

It should be noted that Lima et al. (2001) then set out and colossally apply this grid to *the entire contents of each of the (twelve) textbooks* from the preceding year's PNLD Guide. Their conclusions are rather damning for every single one of them, and we believe that this had a lasting impact in the textbook authors' establishment, and indirectly contributed to raise the national standards of the PNLD itself. To the best of knowledge, this experiment has not been henceforth replicated.

2.2- The Unicamp context

Since its relatively recent foundation in 1966, the University of Campinas (Unicamp) has risen to the position of Latin America's top University⁴. It is public, tuition-free and maintained by São Paulo State; its budget is protected by law at a fixed proportion of the State's yearly tax revenue, currently around R\$ 3 billion or approximately U\$ 1 billion. The community encompasses about 2k faculty and 35k students, of which 19k undergraduates and 16k graduate students. Unicamp is strongly research-oriented, offering 153 postgraduate degrees, relative to 66 at undergraduate level. It responds for about 15% of Brazil's total scientific output, and its patent production is second only to oil giant Petrobrás, among both the public and private sectors.

4 cf. Times Higher Education assessment, 2017.

In Unicamp, the Mathematics *Licenciatura* programme is based at the Institute of Mathematics, Statistics and Scientific Computing (IMECC)⁵, where, for good or worse, it is taught almost exclusively by research mathematicians, and the remaining credits are offered at the School of Education. The degree lasts 8-9 semesters (3120 hours) and it consists mostly of evening classes, seeing as many of the students work on day jobs – quite a few of them actually already teach in some informal capacity. Admissions are subject to the University's dedicated *Vestibular* exam, which covers the whole high school curriculum, weighted by some minor emphasis on the Mathematics score. The junior class typically consists of 70 places, of which some 25-30 eventually graduate every year.

Following changes in Federal and State level pedagogical directives, the degree curriculum underwent a recent reformulation (2013) to incorporate more independent written production and elements of practical experience and cultural diversity. In this context, the formerly optional course on *Analysis of Mathematics textbooks and teaching materials* was also reformed and became mandatory. It is currently offered every other semester on Friday evenings (19:00-23:00) and it accommodates up to 25 participants.

3- Methodology

This article addresses some qualitative conclusions based on three iterations of the course on *Analysis of Mathematics textbooks and teaching materials*, lectured to Unicamp's *Licenciatura* classes by the first-named author. It should be mentioned from the outset that, not unlike most other faculty working with Mathematics *Licenciatura* classes, the lecturer is a research mathematician with virtually no authoritative training in Education. The course evolved therefore as a two-way dialectical learning process, in which students' pedagogical background often took prominence and helped shape the course dynamics, as well as its materials. Indeed, the focus of our investigation lies in the various methodological resources developed during these experiences and it is largely derived from students' own perceptions and independent intellectual production.

In particular, on the issue of comparative analysis, there are very few references describing the process of choice by Mathematics teachers (eg. Rosa, Ribas & Barazzutti, 2012) and one of the aims of this paper is to contribute to this relevant research subject.

Our source of data consists essentially of coursework by students, produced over the years 2014 to 2016 supplemented by oral accounts from students themselves. Since the course itself has been recently created, its approach to pedagogical and methodological questions follows a contemporary perspective, quite often derived from current theoretical trends which help shape the students' pre-existing views of Mathematical Education.

4- Outline of the *Maths textbook analysis* course at Unicamp

4.1- The five tasks, abilities and skills

The course is organised in fifteen weekly workshops of 4h each, divided in five four-week modules (up to overlapping, see below) in which teams of 4-5 students must complete a concrete task, assessed by a written report and an oral presentation of findings discussed with the whole class. The tasks address the following abilities:

Task 1: Vertical analysis - to criticise and improve on a textbook.

Task 2: Horizontal analysis - to compare two textbooks on the same content.

5 Instituto de Matemática, Estatística e Computação Científica, in Portuguese.

Task 3: Foreign textbook analysis - to extract useful resources from foreign materials.

Task 4: Production of teaching material - to write original content to a high standard of text and layout.

Task 5: Original contribution - to take initiative and suggest an independent contribution.

As to secondary skill developments, Tasks 1 to 3 require the students to formulate and apply methodological protocols and assessment grids and Task 4 requires advanced layout management. Having in mind a gradual build-up of template creation skills towards Task 4, all coursework from the outset must be submitted in LaTeX. Students are also encouraged to coordinate asynchronous teamwork combining chat apps, cloud folders and shared editing alternatives.

Each task module iterates a 4-week cycle, organised as follows:

Week 1: Description and motivation.

Week 2: Formatting and discussion of assessment.

Week 3: Critical pre-submission review.

Week 4: Submission of report and oral presentation.

Notice that subsequent modules overlap: the Week 4 activity of Task n and the Week 1 activity of Task $n+1$ use up respectively the first and second halves of the same 4h session.

4.2- Teamwork dynamics

In the course experience at hand, students are *not* allowed to form the groups themselves, and are rather reshuffled after each task in order to maximise, in order of priority, (i) the total number of colleagues with which each one interacts during the whole term, and (ii) the heterogeneity of each group, in terms of skills and motivation, as assessed from results of previous tasks and participation in class. This is particularly justified in a workshop-based course which involves substantial extra-classroom commitment, since one most certainly wants to avoid the *good students* clustering up while everyone else is left dependent on direct intervention by the lecturer. Conversely, students confidentially evaluate each team member after a task is handed in, by attributing one of the following three grades:

Grade 1: team member did their share of the work as agreed / default grade, if left blank.

Grade 1/2: team member underperformed the agreed workload, to the point that other team members had to do some of their work in their stead.

Grade 0: team member did not contribute to the task, to the point that all their agreed workload had to be fulfilled by others.

Individual grades are then computed according to the *mode* of the grades assigned by their team members, and this results in a *multiplier* of the original task grade for the group attributed after marking, so eg. if a group task is given the mark 8.0 (over 10), but a majority of team members decide colleague X contributed only 1/2, then their effective grade for the task will be 4.0. The adoption of mode as a criterion, compared for instance with average, dilutes the effect of personal idiosyncrasies and occasional rivalries. Finally, other fractions could of course be allowed, but this might encourage students to overthink a reaction which after all should be a last resort. In practice, reductions in effective grade have occurred very seldom, about once or twice in a semester, but students have reported that the *possibility* of evaluating their peers in fairness to their output is recomforting.

5- Detailed description and findings from Tasks 1 to 5

5.1- Task 1: Vertical analysis

Groups examine a connected segment of about 40 pages from a given PNLD textbook, typically comprising two related sections. The analysis focuses on the intrinsic characteristics of the material, especially mathematical correctness, language, pedagogical approach, examples and nature of tasks proposed.

There is very little technical bibliography on school textbook analysis by Brazilian authors. For this first task, students follow essentially the approach proposed by Lima (2001), a former research mathematician and prolific author of *higher education* maths textbooks. The manipulation of the textbooks and the reading of chosen extracts from this theoretical reference support a classroom discussion about the main conceptual issues involved in the task. The key points raised typically include:

- what ought to be the main axes of analysis: eg. mathematical rigour, language, layout, exercises etc.
- which methodological criteria to adopt, i.e., what is a "good" or "bad" instance of each of the previously outlined axes; these are usually based on students' own past experiences both as pupils and teachers.
- the issue of assessing exercise sections is more involved, and some specific discussion develops about what is an appropriate balance between direct manipulation and applications, with a recurrent emphasis on how to detect false contextualisation (problems whose narrative pretends to involve modelling whilst in practice disguising mere, and often implausible, manipulation).
- the inevitability of reductionism: the dangers and responsibilities which any such analysis entails.

Ensuing the discussion, groups gather and must formulate independently their own methodology, the outcome of which tends to be a mild variation on Lima et al.'s own assessment scheme, as seen above. Each team member will then uniformly apply those methodological axes and criteria during homework, leading typically to the mapping and sorting of "faults" in the studied excerpt and some mild statistical exploration of the findings towards justifiable qualitative conclusions. Usually, this comes in the form of verdicts, for instance "definitions tend to be sloppy and exercises lack contextualisation" or "language tends to be excessively formal for a 5th grade text". Ideally such conclusions should be thought of as hypothetical constructive feedback for the textbook authors.

In parallel, seeing as many students have no prior contact with LaTeX, the class takes a crash introduction, with the help of a teaching assistant. Their objective in this regard is to be able to contribute autonomously to the group effort by providing their share of the analysis directly in LaTeX code, usually via some online platform like Overleaf or Sharelatex.

5.2- Task 2: Horizontal analysis

Groups compare segments of about 30 pages on the same topic, from two distinct PNLD textbooks. The goal now is to be able to justifiably *choose a winner* among the samples offered, regardless of their intrinsic merit. This task reflects upon a common school teacher experience, since each time a new PNLD list is published, every school *must choose* one collection to adopt based on samples sent by the approved printing houses, and our premise is that if any two

options can be systematically compared, then upon finitely many comparisons teachers should be able to rank the whole list of samples and make a rational informed choice.

Again, there is no ready-to-use bibliography by Brazilian authors to support this important decision. In some sense the only available source are the PNLD guidelines themselves, but then again the decision only takes place among nationally approved collections, hence, under the working assumption that the national assessment is minimally coherent, these guidelines are insufficient by definition. In practice, overworked teachers must make these choices at very short notice and with virtually no procedural guidance, so the process is exposed to anti-pedagogical criteria such as authority, tradition and personal idiosyncrasy.

At Unicamp, we took therefore upon ourselves to establish the basic framework for a content-based comparison grid, which is perfected each year from previous iterations of the task. The main elements are:

- An assessment grid is a point-based system which rewards relative good performance of a textbook over the other about each subtopic or content unit, corresponding roughly to one entry in the official syllabus (eg. sketch of the graph of a hyperbola, or distributivity of integer multiplication).
- Subtopics are surveyed and divided into *even* and *odd*, according to whether they occur in both textbooks or in only one of them, respectively.
- Even subtopics are compared in merit, based on some form of simplified "locally vertical" analysis, and at every instance the winner is awarded some points.
- Odd subtopics are examined per se, and usually assessed as positive, irrelevant or detrimental, being accordingly rewarded or even sometimes punished with negative points.
- A typical methodological pitfall lies in so-called *false odds*: topics which do occur on both textbooks, and therefore are even, but under such different pedagogical guises as to elicit a first impression of being different altogether. For instance, a subtopic may occur as opening motivation for the chapter on one book and as a side remark next to a theorem on the other, and they should be treated as even because in a way or another the teacher can trust that the *syllabus will be covered* in that respect. The resolution and assessment of false odds tend to require quite a bit of interaction between the lecturer and individual groups, as well as some wider classroom sharing of controversial cases.
- Another common pitfall is the temptation to produce *two vertical analyses*, thereby misplacing time and energy towards discussing the individual merits and shortcomings of each book, as opposed to sometimes frustrating actual task of choosing the *least bad* one. This impulse is understandable, since the students feel naturally inclined to apply the skills they just acquired in Task 1, and it must be constantly monitored by the lecturer during progress discussions.

The standard satisfactory report plays out somewhat as a sports match, in which textbooks score or lose points until a winner emerges. The central point of this is the fact that the grid must be established *before* actual analysis occurs, so that assessment derives from universal values and consensual pedagogical decisions, rather than case-by-case subjective preference. As a counterweight, however, students are encouraged to make minor updates to their methodology once, after the second week of the task, to accommodate totally unexpected phenomena and to exclude methodological expectations which did not materialise at all - hence have no purpose for comparison.

5.3- Task 3: Foreign textbook analysis

Students are made aware, in some cases apparently for the first time, that Mathematics is arguably the most universal of school subjects and that its teaching has been addressed from very different world views for centuries. This task consists in examining a 30-page section of a foreign textbook from our library's collection, with the sole and explicit goal of extracting good practices, both in content and in layout, which in their opinion constitute relative innovations and *can be effectively applied* to improve Brazilian textbooks. To state it clearly, it is an exercise in constructive plagiarism and technological catch up.

Currently available materials are from Cuba, France, Japan (in Japanese), Russia (in English) and Spain, mostly obtained by donation from visiting faculty, postdocs etc over time.⁶ Clearly a preference is given by students to textbooks in Latin languages or in English, which can be efficiently read by most, but some have ventured for instance into A. Givental's translation of Kiselev's Planimetry, yielding interesting results (see below).

Groups are asked first of all to do some minor geographic research and establish a comparison between the education systems of the relevant country and Brazil, as well as broader socio-economic factors such as population and per capita income, Gini index, PISA scores and literacy rates. An interesting finding, for example, is that Spain is soundly comparable to São Paulo State in most of these variables, albeit teachers are much better paid, whereas Cuba has much fewer resources but relatively superior results.

Here are some highlighted elements which emerged from this exercise over recent years:

- French and Spanish textbooks tend to have a much more solid chapter and sectioning structure: sections begin with a recap and a statement of goals or skills to be developed, which is then matched by some form of progress assessment at the end. This may seem trivial to non-Brazilians but it is hardly ever correctly implemented, if it is to be found at all, in the Brazilian textbooks surveyed in this course.
- Some Spanish textbooks tend to be explicit as to the difficulty of exercises, or to the specific skill being practiced, relative to the chapter's goals. This is usually done by some visual code involving colours or "difficulty bars"; students have found this to be a time-saver both for the teacher's lesson-planning and for the pupil's independent study.
- Cuban books have relatively poor printing quality, but they tend to be very carefully adjusted to local reality: for instance, the first few sections in a 1st grade primary school textbook are strictly pictorial, since children are meanwhile learning how to read, and written text is gradually introduced along the book with increasingly complex grammar, so as to follow their progress in Spanish along the year. Students assert that this sort of interdisciplinary pedagogical integration is very different from standard practice in Brazilian primary schools.
- Kiselev's 19th Century pedagogy in Geometry is mostly seen as a monstrosity by students' sensitive contemporary eyes. However, one group has made a very interesting use of its rigorous definition-theorem-proof structure and elaborate ruler-and-compass constructions (aimed at 6th graders...), by a clever and rather straightforward adaptation of a sample section into a set of activity sheets for Geogebra.
- Although the assessment criteria in this task are necessarily more flexible, to account for the diversity of materials and outcomes, groups are already well-aware that the next task involves the production of original material, so they tend to take very seriously every hint that might give them a headstart in that near future.

⁶ The reader is very much invited to send us donations from their own country, with our utmost gratitude.

5.4- Task 4: Production of teaching material

Students must apply the skills acquired in Tasks 1-3 into the original production of two sections of a hypothetical textbook, spanning over 20-40 pages, to a high standard of content and layout. While of course this is an opportunity to consolidate their conceptual achievements so far, the task is first and foremost an experience of the passive pole in criticism, a proficiency which tends to quickly hypertrophy in younger generations' analytical culture. Students report that wearing the author's shoes is a humbling experience, and that feeling just how hard it is to put their best into paper tends to inspire a certain benevolence toward textbooks they may have viciously denounced in Tasks 1 and 2.

Each group must choose two related topics, establish a pedagogical approach and create a LaTeX template accommodating all the structural elements required. In particular, they must create dedicated visual language to designate recaps, definitions, examples, curiosities, reminders etc. The A-standard for this material is both (i) to be visually indistinguishable from a commercially available textbook and (ii) to stand on its feet upon subsequent vertical analysis by the lecturer. Here are some of our systematic findings:

- Seeing as the creation of a LaTeX template from scratch (or the major adaptation of something available online), as well as intense decision-making on the selection, phrasing and ordering of content, must happen within just over three weeks, it might seem that this measure is altogether unachievable. However, students tend to surpass all expectations and produce some really outstanding material.
- Successful groups systematically apply lessons learned from foreign books in Task 3 and show a permanent concern not to incur in faults detected in Task 1.
- Successful groups maximise their work capacity by assigning to the most LaTeX-savvy member an exclusively editorial role, essentially creating environments and visual resources upon request from the "contents team" and compiling together their contributions.
- The least able or least motivated students tend to be assigned to online searching for useful images or to browsing existing textbooks for good exercises to borrow; in virtually no observed instances did any team member stand idle.
- Only just about one group each year tends to decisively fail Task 4.

We conclude with some highlighted Task 4 works from each of the course iterations; non-Portuguese speakers may still appreciate the layout and structure choices, as well as some elements of the pedagogical approach:

- 2014: <http://www2.ime.unicamp.br/~ma225/2014Tarefa4-GrupoA.pdf>
This is a 5th grade chapter on Fractions, divided in the two sections *Revision of fractions* and *Operations with fractions*. It demonstrates careful planning of chapter structure, beginning with motivation and statement of goals and following, for each unit, the consistent expository pattern of content - example - practice. The text reflects considerable attention to the adequacy of written and visual language to that specific age group.
- 2015: <http://www2.ime.unicamp.br/~ma225/2015Tarefa4-GrupoB.pdf>
This is an 8th grade chapter on Proportions, divided in the two sections *Proportionality* and *Thales' theorem*. In this beautifully designed template, a particular pedagogical priority is placed on exercises, both as motivation and *a posteriori* practice, and their difficulty is highlighted to allow both teacher and student to gauge their planning and

expectations. At the end of each section, a Summary page is included, just before the exercise sheets, and a Geogebra construction activity is proposed. At the end of the Chapter there is a selection of real questions appearing in Vestibular admissions exams across the country, and a card game is proposed.

- 2016: <http://www2.ime.unicamp.br/~ma225/2016Tarefa4-GrupoC.pdf>
This is a High School 1st year chapter on quadratic polynomials, divided in the two sections *Quadratic functions* and *Inequalities in degree 2*. Although the template is less impressive than the previous two examples, this document reflects appropriately the level of mathematical formalism for this age group, and it reveals a deliberate pedagogical choice to motivate quadratic phenomena from the mechanics of sport, as opposed to say ballistics or stale "free-fall" physics situations. It also offers some graph-plotting activities on Geogebra.

5.5- Task 5: Original contribution

The final task of the course pushes for an independent initiative proposed by the groups themselves, according to two guiding approaches: (i) to extrapolate the immediate scope of the course, or (ii) produce a lasting contribution to further iterations of the course itself. Approach (i) has resulted for instance in the vertical analysis of a *Physics* textbook or the development of software modelling activities to complement an otherwise good textbook, while several interesting outcomes of (ii) are discussed below. Once the task is presented, groups must form a consensus around a proposal, which is then validated into a concrete project with the lecturer's input. The expectations and assessment criteria are then directly negotiated between each group and the lecturer.

Task 5 seeks to stimulate initiative, independence, creativity and generosity to future generations of students and to wider society. These values are well-reflected in many contributions so far, of which we highlight the following:

- The course website: <http://www2.ime.unicamp.br/~ma225/>
- The official webpage for the course was itself an outcome of Task 5 from the 2014 class, entirely designed and developed by the students themselves.
- Course manual: aimed at facilitating future generations' assimilation of the course's rationale and goals, it includes an instructive description of each task, a specific section on LaTeX for textbook templates and a detailed report of the 2014 class experience from the students' perspective.
- Slideshow: the actual big screen supporting material currently by the lecture, containing hyperlinks to the course's webpage, examples from previous years, standards for efficient workshop dynamics during the class and standards for group communication and submission of tasks, among others.
- LaTeX templates: a number of ready-to-use templates for the use of future groups in each of Tasks 1-4, accounting for subtleties such as age group-specific suggestions of visual resources etc.
- Online games inventory: <http://www2.ime.unicamp.br/~ma225/jogos/>
- Conceived as an online support platform for teachers, this compendium of online games is organised according to the national curriculum, so that one can easily find links to online resources sorted by specific curriculum entry. Quality was a big concern, and a methodological decision was made to offer *no link at all* for a given topic if no

pedagogically sound resource could be found for it. The idea of course is to be gradually expanded by future classes.

Afterword

We believe the findings of our qualitative analysis can contribute both to the theoretical field of Mathematics Textbook Analysis and to the practice of school teachers in a number of ways, based on our Brazilian experience. First, by establishing some systematic patterns to anchor the criticism and improvement of a given textbook. Second, by formulating methodological cues to support the decision-making process of teachers among a sample of alternative collections. Third, by highlighting the potential input of textbooks from different cultures and epochs, both in content and in layout, relative to the Brazilian standard. Fourth though not least, to corroborate the perception that future teachers, while still in-training, already display huge creative energy and, albeit not necessarily the strongest students in Higher Mathematics as such, can vastly outperform expectations when provided a collaborative and stimulating classroom environment.

Finally, we hope this account may motivate similar experiences in other teacher-training institutions, which can then be compared and evolve together into a core set of good practices to be adopted, in various guises, on an international scale.

References

BORBA, R.; SELVA, A. 2013. Analysis of the role of the calculator in Brazilian textbooks. *ZDM Mathematics Education*. 45. p.737-750.

BRASIL. *Programa Nacional do Livro Didático (PNLD)*. Ministério da Educação. Available at: http://portal.mec.gov.br/index.php?Itemid=668&id=12391&option=com_content&view=article. Last access: 20 October 2012.

BRASIL. *Pró-letramento – Apresentação*. Ministério da Educação. Available at: http://portal.mec.gov.br/index.php?option=com_content&view=article&id=12346&Itemid=698. Last access: 04 February 2014.

LIMA, E. et al. Exame de Textos: Análise de livros de Matemática para o Ensino Médio, 2001.

MAZZI, L.C.; AMARAL, R.B. Brazil's mathematics textbooks: an overview of the government policies. In: International Conference on Mathematics Textbook Research and Development (ICMT), 2014, Southampton. *Proceedings of ICMT*, 2014.

ROSA, C.P.; RIBAS, L.C.; BARAZZUTTI, M. Análise de livros didáticos. In: Encontro Nacional Pibid-Matemática. *Anais...* 2012.

SALLA, F. PNLD 2013: como escolher livros com critério (PNLD 2013: how to choose textbooks with criterion). *Nova escola*. 251, April 2012.

VALVERDE, G.A.; BIANCHI, L.J.; WOLFE, R.G.; SCHIMDT, W.H.; HOUANG, R.T. *According to the book: using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer. 2002.

