PROVA 1 NOTURNO.

1. $(2y+1)\,dx + \left(\frac{x^2-y}{x}\right)dy = 0.$

$M = 2y+1 \qquad N = \frac{x^2-y}{x} = x - \frac{y}{x}$

$M_y = 2 \neq N_x = 1 + yx^{-2}$ não é exata. 05

$$\frac{M_y - N_x}{N} = \frac{2-1-yx^{-2}}{x - yx^{-1}} = \frac{(1-yx^{-2})}{x(1-yx^{-2})} = \frac{1}{x}$$

$\mu(x) = e^{\int \frac{1}{x}} = e^{\ln x} = x.$ 05

$(2yx + x)\,dx + (x^2 - y)\,dy = 0$

$M_y = 2x = N_x = 2x \Rightarrow$ é exata. 1.0

$\Rightarrow$ Existe $F(x,y) = C$ t.q. $F_x = M$ e $F_y = N$

$F = \int F_x\,dx = \int (2yx + x)\,dx + g(y) =$

$F = \cancel{2}y\frac{x^2}{\cancel{2}} + \frac{x^2}{2} + g(y)$ 05

$F_y = x^2 + g'(y) = N = x^2 - y$

$g'(y) = -y \qquad g(y) = -\frac{y^2}{2}.$ 05

$$\boxed{yx^2 + \frac{x^2}{2} - \frac{y^2}{2} = C}$$ 1.0

2. a) $x^2y'' + 7xy' + 5y = 0$

$y = x^r \qquad y' = rx^{r-1} \qquad y'' = r(r-1)x^{r-2}$

$r(r-1)x^{r-2}\cdot x^2 + 7rx^{r-1}x + 5x^r = 0$

$x^r\left[r(r-1) + 7r + 5\right] = 0$

$(r^2 + 6r + 5) = 0 \qquad \dfrac{-6 \pm \sqrt{36-20}}{2} = \dfrac{-6 \pm 4}{2}$

$r = -5 \qquad r = -1$

$\boxed{y_c = c_1x^{-5} + c_2x^{-1}}$ 1.0

b) $y_p = u_1x^{-5} + u_2x^{-1}$

$x^2y'' + 7xy' + 5y = x$

$\hookrightarrow f(x) = \dfrac{x}{x^2} = \dfrac{1}{x}$

04 $\begin{cases} u_1'x^{-5} + u_2'x^{-1} = 0 \\ -5u_1'x^{-6} - u_2x^{-2} = x^{-1} \end{cases}$

02 $W = \begin{vmatrix} x^{-5} & x^{-1} \\ -5x^{-6} & -x^{-2} \end{vmatrix} = -x^{-7} + 5x^{-7} = 4x^{-7}$

03 $u_1' = \dfrac{\begin{vmatrix} 0 & x^{-1} \\ x^{-1} & -x^{-2} \end{vmatrix}}{4x^{-7}} = \dfrac{-x^{-2}}{4x^{-7}} = -\dfrac{x^5}{4}$

03 $u_2' = \dfrac{\begin{vmatrix} x^{-5} & 0 \\ -5x^{-6} & x^{-1} \end{vmatrix}}{4x^{-7}} = \dfrac{x^{-6}}{4x^{-7}} = \dfrac{1}{4}x$

$u_1 = -\dfrac{x^6}{24} \qquad u_2 = \dfrac{x^2}{8}$

const. de integração podem ser zero.

$y_p = -\dfrac{x^6}{24}\cdot x^{-5} + \dfrac{x^2}{8}x^{-1} = -\dfrac{1}{24}x + \dfrac{1}{8}x = \dfrac{2}{24}x$

03 $\boxed{y_p = \dfrac{x}{12}} \qquad \boxed{y = c_1x^{-5} + c_2x^{-1} + \dfrac{x}{12}}$

3. $yy'' + (y')^2 = 0$. $\quad y > 0$, $y' > 0$

$v(y(x)) = y' \Rightarrow y'' = v'(y(x))\, y'(x) = v'v$ 05

$yv'v + v^2 = 0$

$yv' + v = 0 \Rightarrow yv' = -v$

05 $\displaystyle\int \frac{dv}{v} = \int -\frac{dy}{y} \Rightarrow \ln v = -\ln y + C$.

$y' = v = Cy^{-1} \Rightarrow \int y\,dy = \int C\,dx$

05 $\boxed{\frac{y^2}{2} = Cx + D}$

4. $y'' + 4y' + 5y = 8\delta_3(t) \qquad y(0) = y'(0) = 1$

$s^2 X(s) - s - 1 + 4(sX(s) - 1) + 5X(s) = 8e^{-3s}$

$(\underbrace{s^2 + 4s + 5}_{(s+2)^2 + 1})X(s) = 8e^{-3s} + s + 5$

$\Rightarrow$ 1.0 $\displaystyle X(s) = \frac{8e^{-3s}}{(s+2)^2 + 1} + \frac{s+2}{(s+2)^2+1} + \frac{3}{(s+2)^2+1}$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\} = e^{-2t}\operatorname{sen} t$ 05 $\quad \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} = e^{-2t}\cos t$ 05

$\mathcal{L}^{-1}\left\{e^{-3s} \cdot \frac{1}{(s+2)^2+1}\right\} = u_3(t) f(t-3) = $

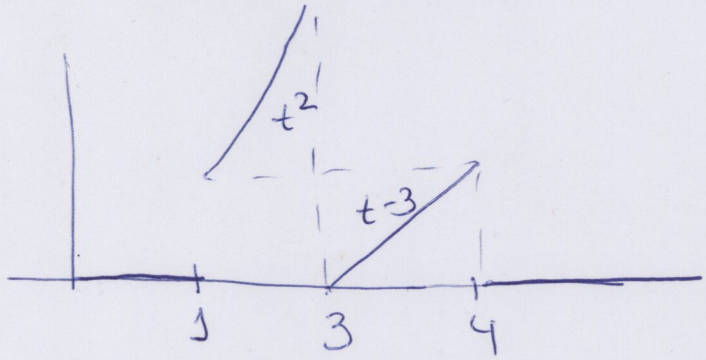
$= u_3(t)\, e^{-2(t-3)} \operatorname{sen}(t-3)$

05

$\mathcal{L}^{-1}\{X(s)\} = \boxed{y(t) = 8u_3(t)e^{-2(t-3)}\operatorname{sen}(t-3) + e^{-2t}\cos t + 3e^{-2t}\operatorname{sen} t}$

5.

a)



$$f(t) = u_1(t)\,t^2 - u_3(t)\,t^2 + u_3(t)(t-3) - u_4(t)(t-3)$$

05

b) $\mathcal{L}\{f(t)\} = \mathcal{L}\{u_1(t)\,t^2\} + \mathcal{L}\{-u_3(t)\,t^2\}$

$+ \mathcal{L}\{u_3(t)(t-3)\} - \mathcal{L}\{u_4(t)(t-3)\}.$

$$= \boxed{e^{-s}\left(\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}\right) - \underbrace{e^{-3s}\left(\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right) + e^{-3s}\left(\frac{1}{s^2}\right)}_{05} - e^{-4s}\left(\frac{1}{s^2}+\frac{1}{s}\right)}$$

02 05 03

pois

a) $\mathcal{L}\{u_1(t)\underbrace{t^2}_{f(t-1)}\} = e^{-s}\mathcal{L}\{f(t)\} = e^{-s}\left[\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}\right]$

$f(t-1) = t^2 \Rightarrow f(t) = (t+1)^2 = t^2+2t+1$

b) $\mathcal{L}\{u_3(t)\underbrace{t^2}_{f(t-3)}\} = e^{-3s}\mathcal{L}\{f(t)\} = e^{-3s}\left[\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right]$

$f(t-3) = t^2 \Rightarrow f(t) = (t+3)^2 = t^2+6t+9.$

c) $\mathcal{L}\{u_3(t)\underbrace{(t-3)}_{f(t-3)}\} = e^{-3s}\frac{1}{s^2}$

d) $\mathcal{L}\{u_4(t)\underbrace{(t-3)}_{f(t-4)}\} = e^{-4s}\mathcal{L}\{t+1\} = e^{-4s}\left(\frac{1}{s^2}+\frac{1}{s}\right)$

$t-4 = v$

$f(v) = v+4-3 = v+1$