(Purely) coclosed G_2-structures on 2-step nilmanifolds

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In Riemannian geometry, simply connected nilpotent Lie groups endowed with left-invariant metrics, and their compact quotients, have been the source of valuable examples in the field. This motivated several authors to study, in particular, left-invariant G_2-structures on 7-dimensional nilpotent Lie groups. These structures could also be induced to the associated compact quotients, also known as *nilmanifolds*.

Left-invariant torsion free G_2-structures, that is, defined by a simultaneously closed and coclosed positive 3-form, do not exist on nilpotent Lie groups. But relaxations of this condition have been the subject of study on nilmanifolds lately. One of them are coclosed G_2-structures, for which the defining 3-form verifies $\mathrm{d} \star_{g_\varphi} \varphi = 0$, and more specifically, purely coclosed structures, which are defined as those which are coclosed and satisfy $\varphi \wedge \mathrm{d}\varphi = 0$.

In this talk, there will be presented recent classification results regarding left-invariant coclosed and purely coclosed G_2-structures on 2-step nilpotent Lie groups. Our techniques exploit the correspondence between left-invariant tensors on the Lie group and their linear analogues at the Lie algebra level. In particular, left-invariant G_2-structures on a Lie group will be seen as alternating trilinear forms defined on the Lie algebra. The coclosed condition now refers to the Chevalley-Eilenberg differential of the Lie algebra. We also rely on the particular Lie algebraic structure of metric 2-step nilpotent Lie algebras.

Our goals are twofold. On the one hand we give the isomorphism classes of 2-step nilpotent Lie algebras admitting purely coclosed G_2-structures. The analogous result for coclosed structures was obtained by Bagaglini, Fernández and Fino [Forum Math. 2018].

On the other hand, we focus on the question of *which metrics* on these Lie algebras can be induced by a coclosed or purely coclosed structure. We show that any left-invariant metric is induced by a coclosed structure, whereas every Lie algebra admitting purely coclosed structures admits metrics which are not induced by any such a structure. In the way of proving these results we obtain a method to construct purely coclosed G_2-structures. As a consequence, we obtain new examples of compact nilmanifolds carrying purely coclosed G_2-structures.