Gap theorems for free-boundary submanifolds

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Abstract: Let M^n be a compact n-dimensional manifold minimally immersed in a unit sphere S^{n+k} and let denote by $|A|^2$ the squared norm of its second fundamental form. It follows from the famous Simons pinching theorem that if $|A|^2 \leq \frac{n}{2-\frac{1}{k}}$, then either $|A|^2 = 0$ or $|A|^2 = \frac{n}{2-\frac{1}{k}}$. The submanifolds on which $|A|^2 = \frac{n}{2-\frac{1}{k}}$ were characterized by Lawson (when $k = 1$) and by Chern-do Carmo-Kobayashi (for any k).
These important results say that there exists a gap in the space of minimal submanifolds in S^{n+k} in terms of the length of their second fundamental forms and their dimensions.
Latter, Lawson and Simons proved a topological gap result without making any assumption on the mean curvature of the submanifold. Namely, they proved that if M^n is a compact submanifold in S^{n+k} such that $|A|^2 \leq \min\{p(n-p), 2\sqrt{p(n-p)}\}$, then for any finitely generated Abelian group G, $H_p(M;G) = 0$. In particular, if $|A|^2 < \min\{n-1, 2\sqrt{n-1}\}$, then M is a homotopy sphere.
It is well known that free-boundary minimal submanifolds in the unit ball share similar properties as compact minimal submanifolds in the round sphere. For instance, Ambrozio and Nunes obtained a geometric gap type theorem for free-boundary minimal surfaces M in the Euclidean unit 3-ball B^3. They proved that if $|A|^2(x)\langle x, N(x)\rangle^2 \leq 2$, where $N(x)$ is the unit normal vector at $x \in M$, then M is either the equatorial disk or the critical catenoid.
In the first part of this talk, I will present a generalization of Ambrozio and Nunes theorem for constant mean curvature surfaces. Precisely, if the traceless second fundamental form ϕ of a free-boundary CMC surface B^3 satisfies $|\phi|^2(x)\langle x, N(x)\rangle^2 \leq (2 + H\langle x, N(x)\rangle)^2/2$ then M is either a spherical cap or a portion of a Delaunay surface. This is joint work with Barbosa and Pereira.
In the second part, I will present a topological gap theorem for free-boundary submanifolds in the unit ball. More precisely, if $|\phi|^2 \leq \frac{np}{n-p}$, then the p-th cohomology group of M with real coefficients vanishes. In particular, if $|\phi|^2 \leq \frac{n}{n-1}$, then M has only one boundary component. This is joint work with Mendes and Vitório.