

Universidade Estadual de Campinas
Instituto de Matemática, Estatística e Computação Científica

IV Escola Brasileira de Sistemas Dinâmicos

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Caderno de Resumos

Apoio:



Evento realizado pelo IMECC/Unicamp na cidade de
Valinhos/SP

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Sumário

Mini-cursos	v
Single orbit dynamics (<i>Dominik Kwietniak</i>)	v
Variedades Descontinuamente Folheadas (<i>Paulo Ricardo da Silva</i>)	v
Homologia de contato e a estrutura variacional por trás dos fluxos de Reeb (<i>Pedro Salomão e Umberto Hryniewicz</i>)	vi
Emergence and Para-Dynamics (<i>Pierre Berger</i>)	vi
 Palestras	 vii
Systolic inequalities and Reeb dynamics (<i>Alberto Abbondandolo</i>)	vii
Aplicações Induzidas (<i>Alexander Arbieto</i>)	vii
Dynamics of linear operators (<i>Ali Messaoudi</i>)	vii
Tangências e Sistemas Iterados (<i>Artem Raibekas</i>)	viii
Geometria fractal e bifurcações homoclínicas (<i>Carlos Gustavo T. de A. Moreira</i>)	viii
Generic Bifurcations in Refractive Systems (<i>Claudio Buzzi</i>)	viii
Generalized Weierstrass functions, linear response problem and central li- mit theorem (<i>Daniel Smania</i>)	ix
Dynamics near Bonatti-Diaz cycles (<i>Dmitry Turaev</i>)	ix
Chaos induced by sliding phenomena in Filippov systems (<i>Douglas Novaes</i>)	ix
Fuzzy attractors of Iterated Function Systems (<i>Elismar Oliveira</i>)	ix
Conley pairs and geometry - some examples (<i>Joachim Weber</i>)	x
Connections between Mathematical Statistical Mechanics and Symbolic Dynamics (<i>Leandro Cioletti</i>)	x
Reeb flows with positive topological entropy and connected sums (<i>Leo- nardo Macarini</i>)	xi
Transitive nonhyperbolic step skew-products: Ergodic approximation and entropy spectrum of Lyapunov exponents. (<i>Lorenzo Díaz</i>)	xi
Moment Lyapunov Exponents for i.i.d. Random Products and Semigroups (<i>Luiz A. B. San Martin</i>)	xii
On the Lyapunov spectrum of relative transfer operators (<i>Manuel Stadlbauer</i>)	xiv
Growth of Sobolev norms in the nonlinear Schrödinger equation (<i>Marcel Guardia</i>)	xiv
Mean action and the Calabi invariant (<i>Michael Hutchings</i>)	xiv
Contact geometry of the unbounded component in RTBP (<i>Otto Van Koert</i>)	xv
Ergodicity for higher dimensional cylindrical cascade (<i>Patrícia Cirilo</i>) . . .	xv

The existence and structure of generalized Reeb components in codimension one foliations - a generalization of Novikov's Theorem (<i>Paul Schweitzer</i>)	xv
On the ergodic theory of free semigroup actions (<i>Paulo Varandas</i>)	xvi
Partial hyperbolicity and foliations in 3-manifolds (<i>Rafael Potrie</i>)	xvi
Free Seifert fibered pieces of pseudo-Anosov flows (<i>Sergio Fenley</i>)	xvii
Regularizations in non-smooth systems (<i>Tere Seara</i>)	xvii
A link between Topological Entropy and Lyapunov Exponents (<i>Thiago Catalan</i>)	xvii
Anosov Representations and Lorentzian Geometry (<i>Thierry Barbot</i>)	xviii
Periodic orbits in the restricted three body problem and Arnold's J^+ invariant. (<i>Urs Frauenfelder</i>)	xviii
Topological ergodicity and pointwise entropy (<i>Vilton Pinheiro</i>)	xviii
Symplectic embeddings, toric domains and billiards (<i>Vinicio Gripp Barros Ramos</i>)	xix
Rigidity of smooth critical circle maps (<i>Welington de Melo</i>)	xix

Sessão de Poster	xxi
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Programação

Programação IV EBSD

	Seg	Ter	Qua	Qui	Sex
8:45-9:35	Inscrições	Berger	Berger	Berger	Berger
9:40-10:30	Abbondandolo	Umberto/Pedro	Umberto/Pedro	Umberto/Pedro	Umberto/Pedro
10:30-11	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00-11:50	San Martin	Turaev	Hutchings	L. Díaz	Seara
11:55-12:45	Fenley / Stadlbauer	Dominik	Dominik	Dominik	Dominik
12:45-14:30	Almoço	Almoço	Almoço	Almoço	Almoço
14:30-15:20	V. Ramos / Smania	de Melo	Varandas / Guardia	Macarini / Messaoudi	Arbieto / Novaes
15:25-16:15	E. Oliveira / Potrie	Gugu	Cirilo / Frauenfelder	Cioletti / Buzzi	Barbot / Raibekas
16:15-16:45	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
16:45-17:35	Pinheiro / van Koert	Joa / Catalan	Poster	Poster	Schweitzer
17:40-18:30	P. Ricardo	P. Ricardo	P. Ricardo	P. Ricardo	

Evento social
(a partir de 19:30)

Mini-cursos

Single orbit dynamics

Dominik Kwietniak

Jagiellonian University and Universidade Federal do Rio de Janeiro

Single orbit dynamics studies interactions between properties of a dynamical system and behavior of distinguished orbits of that system. Crudely, we try to answer the following question: How much knowledge of a system can be recovered from the existence of an individual orbit with certain properties? This point of view is implicit in a large part of topological dynamics and ergodic theory. It is related to such notions as: generic points, invariant measures, specification, shadowing and heredity. During the lectures we will discuss some old and new results lying at the intersection of topological dynamics and ergodic theory.

Variedades Descontinuamente Folheadas

Paulo Ricardo da Silva

IBILCE-UNESP

A finite set of C^r -vector fields $x \mapsto X_i(x)$, $x \in \overline{\mathcal{U}_i}$, defines a piecewise-smooth vector field X on a n -dimensional manifold M with $M = \bigcup \overline{\mathcal{U}_i}$, \mathcal{U}_i open and $\mathcal{U}_i \cap \mathcal{U}_j = \emptyset$. The switching set is $\Sigma = \bigcup \partial \mathcal{U}_i$. A regularization of X is a 1-parameter family X^ε of C^r vector fields, $r \geq 1$, satisfying that $X^\varepsilon(p) \rightarrow X(p)$, for any $p \in M \setminus \Sigma$, when $\varepsilon \rightarrow 0^+$. We study regularization processes when the switching set is a regular surface and when its singular set is not empty. We introduce the concept of discontinuously foliated manifold and regularization of piecewise smooth oriented 1-foliation. The regularization processes studied are of the kinds Filippov, C^r -blow-up smoothable and of the kind transition. We show that the regularization of the kind transition gives a manifold on which the singular fiber Σ_0 has an invariant manifold \mathcal{S} . The reduced flow on \mathcal{S} is equivalent to the sliding flow when \mathcal{S} is normally hyperbolic. The case where \mathcal{S} it is not normally hyperbolic appears when the regularization is not of the kind Filippov.

Homologia de contato e a estrutura variacional por trás dos fluxos de Reeb

Pedro Salomão¹ e Umberto Hryniewicz²

IME-USP¹, UFRJ²

No vasto universo dos níveis de energia destacam-se aqueles com tipo de contato por apresentarem uma rica estrutura variacional. Em diversos casos se pode definir uma espécie de homologia de Morse associada ao funcional de ação restrito a tais níveis, chamada de homologia de contato. A homologia de contato pode ser vista como uma variação da homologia de Floer, onde se prescreve a energia ao invés do período. Neste mini-curso apresentaremos as definições básicas da homologia de contato (em casos onde ela de fato está bem-definida) e, como consequência, daremos diversas aplicações a resultados de existência-multiplicidade de órbitas periódicas.

Emergence and Para-Dynamics

Pierre Berger

Université Paris 13

Recently we realized that some degenerate bifurcations can occur robustly in the space of dynamical systems. Such a phenomena enables to prove that some pathological dynamics are not negligible and even typical in the sense of Arnold-Kolmogorov.

We will introduce the concept of Emergence which quantifies how wild is the dynamics from the statistical viewpoint, and we will conjecture the local typicality of super-polynomial ones with the space of dynamical systems.

We will focus on one example of Para-Dynamics, by giving a negative answer to the following problem of Arnold (1989): There exists a locally topologically generic set of families $(f_a)_a$ of C^r dynamics f_a , so that for every a , the map f_a has a fast increasing of periodic points:

$$\limsup \log \text{Card Per}_n f_a / n = \infty$$

This will be the occasion to visit several topics in dynamics:

- Para-bifurcation and parablenders,
 - Renormalization in the presence of an hetero-dimensional cycle,
 - Hirsh-Pugh-Shub theory,
 - Parabolic bifurcation for parameter family,
 - KAM theory.
-

Palestras

Systolic inequalities and Reeb dynamics

Alberto Abbondandolo
Ruhr Universität Bochum

A number of questions in systolic geometry, convex geometry and symplectic topology can be reformulated in terms of inequalities involving the minimal action of closed Reeb orbits and the contact volume. I will discuss these connections, together with some recent results which are part of a joint work with B. Bramham, U. Hryniewicz and P. Salomão.

Aplicações Induzidas

Alexander Arbieto
UFRJ

Dada uma dinâmica sobre uma variedade, temos dinâmicas induzidas naturais. A ação sobre o espaço de compactos da variedade (dito o hiperespaço) e também a ação sobre o espaço de probabilidades (dita o push-forward). Recentemente diversos trabalhos versam sobre as relações entre propriedades dinâmicas destas dinâmicas induzidas e propriedades similares da base. Daremos um apanhado sobre certos resultados. Também há interesse natural sobre a dinâmica induzida no hiperespaço continuum (o de compactos conexos). Apresentamos certos resultados sobre tal estudo. Em colaboração com Jennyffer Smith.

Dynamics of linear operators

Ali Messaoudi
IBILCE-UNESP

In this work, we study dynamical properties of linear operators acting on Banach space. We establish relationships between notions of expansivity, trasitivity, supercyclicity and shadowing. We also solve a basic problem in linear dynamics by proving the existence of non-hyperbolic invertible operators with the shadowing property. This is a joint work with N. C. Bernardes, P. R. Cirilo, U.B. Darji and E. Pujals.

Tangências e Sistemas Iterados

Artem Raibekas

UFF

Explicarei como construir tangências robustas de codimensão maior que um em conjuntos parcialmente hiperbólicos. Para fazer isso, utilizamos a dinâmica de certos sistemas iterados em fibrados Grassmanianos.

Geometria fractal e bifurações homoclínicas

Carlos Gustavo T. de A. Moreira

IMPA

Generic Bifurcations in Refractive Systems

Claudio Buzzi

UNESP

In this work we study (germs of) piecewise-smooth system on $\mathbb{R}^3, 0$ and $\Sigma \subset \mathbb{R}^3$ be given by $\Sigma = f^{-1}(0)$, where f is (a germ of) a smooth function $f : \mathbb{R}^3, 0 \rightarrow \mathbb{R}$, $f(0) = 0$, having $0 \in \mathbb{R}$ as a regular value (i.e. $\nabla f(p) \neq 0$, for any $p \in f^{-1}(0)$). Call Ω^r the space of vector fields $Z : \mathbb{R}^3, 0 \rightarrow \mathbb{R}^3$ such that

$$Z(x, y, z) = \begin{cases} X(x, y, z), & \text{for } (x, y, z) \in \Sigma_+, \\ Y(x, y, z), & \text{for } (x, y, z) \in \Sigma_-. \end{cases} \quad (1)$$

We write $Z = (X, Y) \in \chi^r \times \chi^r = \Omega^r$. Endow Ω^r with the product topology. We use the notation $Xf(p) = \langle \nabla f(p), X(p) \rangle$. We are interested in the study of discontinuous systems having the property $Xf(p) = Yf(p)$ for all $p \in \Sigma$. These systems are known as *refractive systems*. This class of systems appears in a lot of applications, for example in Relay Systems. We will present the structural stable cases and the generic bifurcations. Our approach is geometrical and we will use invariant foliations to the flow. This work is co-authored by Marco A. Teixeira and João C. Medrado.

Generalized Weierstrass functions, linear response problem and central limit theorem

Daniel Smania
ICMC-USP

Let f be a smooth expanding map of the circle and ν be a smooth real function of the circle. Consider the twisted cohomological equation $v(x) = A(f(x)) - Df(x)A(x)$ which has a unique bounded solution A . We prove that A is either smooth or nowhere differentiable, and if A is nowhere differentiable then the newton quotients of A , after an appropriated normalization, converges in distribution to the normal distribution, with respect to the unique absolutely continuous invariant probability of f . We also show that a similar phenomenon occurs when we study the linear response problem for piecewise expanding unimodal maps. This is a joint work with Amanda de Lima, based on her Ph. D. Thesis at ICMC-USP.

Dynamics near Bonatti-Diaz cycles

Dmitry Turaev
Imperial College London

We will discuss the dynamical behavior of certain systems in regions near Bonatti-Diaz cycles.

Chaos induced by sliding phenomena in Filippov systems

Douglas Novaes
UNICAMP

In this talk we provide a full topological and ergodic description of the dynamics of Filippov systems nearby a sliding Shilnikov orbit Γ . More specifically we prove that the first return map, defined nearby Γ , is topologically conjugate to a Bernoulli shift with infinite topological entropy. In particular, we see that for each $m \in \mathbb{N}$ it has infinitely many periodic points with period m .

Fuzzy attractors of Iterated Function Systems

Elismar Oliveira
UFRGS

In this presentation we will give a brief introduction to Fuzzy sets and fractal attractors of Fuzzy Iterated Function Systems (IFZS). Additionally, we address the question of how to apply this ideas to the recent theory of Generalized IFS (GIFS) developed in the last few years.

Conley pairs and geometry - some examples

Joachim Weber

UNICAMP

We shall advocate the use of certain Conley pairs associated to non-degenerate, or at least isolated, critical points as effective tools in geometry. Applications include the construction of Morse filtrations associated to semi-flows, a new proof of the cell attachment theorem in Morse theory, and a constructive proof of the folklore lemma that there is an open contractible thickening of the unstable manifold of an isolated critical point. The latter immediately implies the Lusternik-Schnirelmann category theorem.

Connections between Mathematical Statistical Mechanics and Symbolic Dynamics

Leandro Cioletti

UNB

In this lecture will be presented some recent results regarding a newfound connection, between Mathematical Statistical Mechanics and Symbolic Dynamics. We initially focus on the dynamics given by the left shift map acting on an infinite cartesian product of a general compact metric space M and the Ruelle operator acting on the space of the continuous functions. In the sequel we explain how and why to extend the Ruelle operator to the space of integrable functions (with respect to the eigenprobabilities). A large family of continuous potentials having only measurable eigenfunctions will be introduced, in order to explain why the Lebesgue space is a natural space to consider when one is looking for eigenfunctions of continuous potentials. We also present results regarding existence of eigenfunctions for Ruelle operator for potentials not satisfying the so-called Bowen's condition. We discuss briefly the main ideas to obtain these eigenfunctions in such generality by introducing a technique we discovered related to sub/super solution to the eigenfunction problem. We will talk about when a regularization procedure can be carried forward and also sketch a line of attack to an old conjecture by Peter Walters. At the end of this talk we explain a probabilistic method used to obtain the uniqueness of the eigenfunction of the Ruelle operator when the alphabet is a general compact metric space. This method is based on the Dobrushin - Lanford - Ruelle theory for Gibbs measures and it provides, among other things, a non trivial generalization of the Bowen's condition. We finalize the talk proving that the set of eigenprobabilities of the dual of the Ruelle operator for any continuous potential is given by the set of the DLR-Gibbs measures determined by a suitable quasi local specification explicitly determined by the potential on the one-dimensional lattice. The content of this lecture is based on the works developed in the last five years in collaboration with Artur Lopes.

Reeb flows with positive topological entropy and connected sums

Leonardo Macarini

UFRJ

Some contact manifolds have the property that the Reeb flow of every contact form supporting the corresponding contact structure has positive topological entropy. Examples of such manifolds are given by the unit sphere bundle of rationally hyperbolic manifolds and manifolds whose fundamental group has exponential growth. In this talk I will discuss the construction of new examples using contact connected sums. If time allows, I will briefly explain how the idea of this construction leads us to the development of a Lagrangian Floer homology on the complement of certain codimension two invariant submanifolds in cotangent bundles. This is joint work with Marcelo Alves.

Transitive nonhyperbolic step skew-products: Ergodic approximation and entropy spectrum of Lyapunov exponents.

Lorenzo Díaz

PUC-Rio

We study transitive nonhyperbolic (having ergodic measures with positive, negative, and zero exponents) step skew-product maps over a shift with C^1 circle diffeomorphisms as fiber maps. For that we introduce a set of axioms capturing the mechanisms of the dynamics of nonhyperbolic robustly transitive systems.

We first prove that nonhyperbolic ergodic measures can be approximated in the weak* topology and in entropy by hyperbolic ones. Thereafter we derive a multi-fractal analysis for the topological entropy of the level sets of Lyapunov exponent. The results are formulated in terms of Legendre-Fenchel transforms of restricted variational pressures.

This talk is based on two join works with K. Gelfert (UFRJ) and M. Rams (IMPAM).

Moment Lyapunov Exponents for i.i.d. Random Products and Semigroups

Luiz A. B. San Martin

UNICAMP

We consider an independent and identically distributed (i.i.d.) random sequence y_n in a semi-simple Lie group G with common law μ and form its random product $g_n = y_n \cdots y_1$. (For example if $G = \mathrm{Sl}(d, \mathbb{R})$ then g_n is a product of random matrices.) The purpose is to describe geometric properties of the semigroup S_μ generated by the support of μ via the asymptotics of the random product g_n .

The asymptotic properties of g_n are described by limits of cocycles $\rho_\lambda(g, x)$ over the flag manifolds. These cocycles are defined after the Iwasawa decomposition $G = KAN$ through the function $\rho : G \times K \rightarrow A$ given by $gu = v\rho(g, u)n$ with $v \in K$, $\rho(g, x) \in A$ and $n \in N$ and the parameter λ is a linear map on $\mathfrak{a} = \log A$.

The Lyapunov exponents of the random product g_n are defined by

$$\Lambda_\lambda(x) = \lim_{n \rightarrow \infty} \frac{1}{n} \log \rho_\lambda(g_n, x).$$

In case $G = \mathrm{Sl}(d, \mathbb{R})$ then \mathfrak{a} is the space of diagonal matrices and if $\lambda_1(D)$ is the first eigenvalue of the diagonal matrix D then $\rho_{\lambda_1}(g, x)$ is the cocycle $\|gv\| / \|v\|$ over the projective space whose Lyapunov exponents are given by the multiplicative ergodic theorem. The same way the cocycles $\rho_{\lambda_1 + \dots + \lambda_k}(g, x)$ over the Grassmannians yield sums of Lyapunov exponents.

The moment Lyapunov exponent of the random product depending on $\lambda \in \mathfrak{a}^*$ and x is defined by

$$\begin{aligned} \gamma_\lambda(x) &= \limsup_{n \rightarrow \infty} \frac{1}{n} \log \int \rho_\lambda(g, x) \mu^{*n}(dg) \\ &= \limsup_{n \rightarrow \infty} \frac{1}{n} \log \mathbb{E}[\rho_\lambda(g_n, x)] \end{aligned}$$

where μ^{*n} is the n -th convolution power of μ .

We assume that $\mathrm{int}S_\mu \neq \emptyset$ (that is, μ is an *étalée* measure). The so-called flag type of S_μ is a flag manifold associated to it that reveals several geometric and algebraic properties of the semigroup (for instance the Jordan form of its elements).

We relate the flag type of S_μ with the behavior as $p \rightarrow -\infty$ of the functions $p \mapsto \gamma_{p\lambda}(x)$. The point is that for the interesting values of λ , $\frac{1}{n} \log \rho_\lambda(g_n, x)$ converges a.s. to a constant (“top” Lyapunov exponent) which is positive. Hence the behavior of $\mathbb{E}[\rho_\lambda(g_n, x)^p]$ for large $p < 0$ tells if there are “late comers” to the limit of $\frac{1}{n} \log \rho_\lambda(g_n, x)$ showing the location of g_n in G .

The moment Lyapunov exponent $\gamma_\lambda(x)$ is related to the spectral radius of the operator

$$(U_\lambda(\mu)f)(x) = \int_G \rho_\lambda(g, x) f(gx) \mu(dg)$$

acting in spaces of continuous functions. The good properties of $\gamma_\lambda(x)$ are obtained via the perturbation theory of these operators.

An stochastic differential equation

$$dg = X(g) dt + \sum_{j=1}^m Y_j(g) \circ dW_j$$

on G (where X and Y_j are right invariant vector fields) yields a one-parameter semigroup μ_t of probability measures. When the results are applied to the measures μ_t the moment Lyapunov exponents are related to the principal eigenvalue of the differential operator

$$L_\lambda^\Theta = \tilde{L} + \frac{1}{2} \sum_{j=1}^m \lambda(q_{Y_j}) \tilde{Y}_j + \lambda(q_X) + \frac{1}{2} \sum_{j=1}^m \lambda(r_{Y_j}) + \frac{1}{2} \sum_{j=1}^m (\lambda(q_{Y_j}))^2$$

where $\tilde{L} + \frac{1}{2} \sum_{j=1}^m \lambda(q_{Y_j}) \tilde{Y}_j$ is a second order hyperbolic differential operator and the other terms are functions defined from the vector fields X and Y_j .

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On the Lyapunov spectrum of relative transfer operators

Manuel Stadlbauer

UFRJ

We study finite dimensional approximations of relative Ruelle operators associated with a skew product $T : (\omega, x) \mapsto (\theta(\omega), \sigma_\omega(x))$ whose base θ is an ergodic automorphism and whose fibers σ_ω are Markov maps with full branches. We prove that these operators can be approximated in the C^0 -topology by positive matrices with an associated dominated splitting. The proof relies on two principal arguments: the dominated splittings are obtained through an extension of a theorem of Bochi-Viana on accessible cocycles, whereas the approximation in the C^0 -topology lead to a new type of convergence theorems for quotients of transfer operators. Namely, we obtained exponential convergence of these quotiens even without invariant measures, which is a result of independent interest. This gave rise to an effective control of the eigenfunctions of the operators and, in particular, to consider an approximation based on stochastic matrizes. Joint work with Mário Bessa (U. Porto, Portugal).

Growth of Sobolev norms in the nonlinear Schrödinger equation

Marcel Guardia

UPC

The study of solutions of Hamiltonian PDEs undergoing growth of Sobolev norms as time tends to infinity has drawn considerable attention in recent years. The importance of growth of Sobolev norms is that it implies that the solution transfers energy to higher modes as time evolves. Consider the defocusing nonlinear Schrödinger equation on the two dimensional torus. Fix $s > 1$. I will report on recent results in which, applying dynamical systems techniques, we show arbitrarily large finite growth and we provide estimates for the time needed to attain this growth.

Mean action and the Calabi invariant

Michael Hutchings

University of California, Berkeley

Given an area-preserving diffeomorphism of the closed unit disk which is a rotation near the boundary, one can naturally define an “action” function on the disk which agrees with the rotation number on the boundary. The Calabi invariant of the diffeomorphism is the average of the action function over the disk. We show that if the Calabi invariant is less than the boundary rotation number, then the infimum over periodic orbits of the average of the action over the periodic orbit is less than or equal to the Calabi invariant. This result is closely related to the dynamics of Reeb vector fields on the three-sphere. The proof uses a new filtration on the embedded contact homology of the three-sphere determined by a transverse knot.

Contact geometry of the unbounded component in RTBP

Otto Van Koert

Seoul National University

We discuss the unbounded component in the restricted three-body problem, and describe a way to compactify it to a contact manifold. This involves a controlled modification of the dynamics at infinity. It turns out that we will always lose dynamical convexity, but we still have some control, and it is possible to construct a system of global surfaces of section for small mass ratio. The contact geometry of this problem will play a role when trying to apply holomorphic curves to RTBP above the second Lagrange point.ă

Ergodicity for higher dimensional cylindrical cascade

Patrícia Cirilo

UNIFESP

Consider a non compact extension of a toral translation as following

$$\begin{aligned} F_{\alpha,\phi} := \quad \mathbb{T}^k \times \mathbb{R}^d &\rightarrow \quad \mathbb{T}^k \times \mathbb{R}^d \\ (x, r) &\mapsto (x + \alpha, r + \phi(x)). \end{aligned}$$

Chevallier and Conze gave examples of recurrent and non-recurrent cocycles in \mathbb{R}^d over rotations of the torus \mathbb{T}^2 . It is natural to ask about ergodicity of such cocycles. In this talk we study ergodicity of maps of type $F_{\alpha,\phi}$. We show that the generic higher dimensional cylindrical cascade (i.e, a skew product in $\mathbb{T}^k \times \mathbb{R}^d$, as $F_{\alpha,\phi}$) above a Liouville translation is ergodic. This is a joint work with Bassam Fayad (Paris 7-IMJ).

The existence and structure of generalized Reeb components in codimension one foliations - a generalization of Novikov's Theorem

Paul Schweitzer

PUC-Rio

A generalized Reeb component with connected boundary is a compact connected manifold with a codimension one foliation such that the boundary is a leaf and the interior fibers over the circle with the leaves as fibers.

A famous theorem of S.P. Novikov states that every foliation of the three-sphere and many other 3-manifolds must contain a classical Reeb component, i.e., a foliated solid torus with topological planes as the interior leaves. First, Novikov shows the existence of a so-called (homotopical) vanishing cycle, i.e., a loop on a leaf that is not contractible on its leaf but can be lifted to loops on nearby leaves where it becomes contractible. Then he shows that a vanishing cycle must lie on the boundary of a Reeb component.

We study the structure and existence of generalized Reeb components and generalize the second half of the proof by showing that a (homological) $(m - 2)$ -dimensional C^1 vanishing cycle satisfying a certain additional condition on the double points in a C^1 oriented codimension one foliation F of a closed oriented m -manifold M lies on the boundary of a generalized Reeb component. As an application, we note that in many generalized Reeb components any flow transverse to the foliation must have a periodic orbit.

This is joint work with Fernando Alcalde and Gilbert Hector over the past twenty years.

On the ergodic theory of free semigroup actions

Paulo Varandas
UFBA

One of the main purposes of dynamical systems is to understand the behavior of the space of orbits of continuous group and semigroup actions on compact metric spaces. The most studied and well understood classes of such dynamical systems are $\mathbb{Z}-$, $\mathbb{N}-$ or \mathbb{R} -actions, which correspond to the dynamics of homeomorphisms, continuous endomorphisms or continuous flows, respectively. A notion of topological complexity for such dynamical systems has been proposed in the seventies and was very well studied by Goodwin, Bowen, Walters and Parry, among others, and proved a variational principle for the pressure. Such strong relations between the topological and ergodic features of a dynamical system is still unavailable for general group actions. On the one hand, the theory is not unified since several notions of topological complexity have been proposed, and many of them depend on properties of the group action as commutativity or amenability. On the other hand, many group actions admit no common invariant measures and this notion should be replaced by a more flexible concept. In this talk I will discuss a notion of topological entropy and pressure for finitely generated semigroup actions and discuss its regularity, and discuss on ergodic aspects of (finitely generated) free semigroup actions. These results are part of joint works with F. Rodrigues (UFRGS, Brazil) and M. Carvalho (U. Porto, Portugal).

Partial hyperbolicity and foliations in 3-manifolds

Rafael Potrie
UdelaR

I will try to review the use of foliation theory in the classification problem of partially hyperbolic diffeomorphisms in 3-manifolds explaining some recent advances in the field towards a better understanding of these systems. In particular, explain a joint work in progress with T. Barthelme, S. Fenley and S. Frankel about classification of partially hyperbolic diffeomorphisms homotopic to identity in 3-manifolds.

Free Seifert fibered pieces of pseudo-Anosov flows

Sergio Fenley

Florida State University, USA

We prove a structure theorem for pseudo-Anosov flows restricted to Seifert fibered pieces of three manifolds. The piece is called periodic if there is a Seifert fibration so that a regular fiber is freely homotopic, up to powers, to a closed orbit of the flow. A non periodic Seifert fibered piece is called free. In this talk we consider free Seifert pieces. We show that, in a carefully defined neighborhood of the free piece, the pseudo-Anosov flow is orbitally equivalent to a hyperbolic blow up of a geodesic flow piece. A geodesic flow piece is a finite cover of the geodesic flow on a compact hyperbolic surface, usually with boundary. We introduce almost k -convergence groups, and an associated convergence theorem. We also introduce an alternative model for the geodesic flow of a hyperbolic surface that is suitable to prove these results, and we define what is a hyperbolic blow up. This is a joint work with Thierry Barbot.

Regularizations in non-smooth systems

Tere Seara

UPC

In this talk we will discuss recent results concerning regularization technics in non-smooth dynamical systems.

A link between Topological Entropy and Lyapunov Exponents

Thiago Catalan

UFU

An important problem in the theory of Dynamical System is how to measure/compare the complexity of different dynamics. The most famous tool, in this sense, is the topological entropy, $h_{top}(f)$. This invariant concept measures the rate of exponential growth of the number of distinct orbits when we compare orbits with finite length. Other way to measure the complexity of dynamics is using Lyapunov Exponents. This number tell us the rate of divergence of nearby trajectories. Lyapunov Exponents measure the asymptotic behavior of dynamics in the tangent space level. The main result we want to present links topological entropy and Lyapunov exponents in the symplectic setting.

To be more precisely, let M be a symplectic manifold. Given a C^1 -diffeomorphism f on M , we define $S(f)$ as been the supremum of the sum of the positive Lyapunov exponents of the hyperbolic periodic points of f . Then we prove that there exists a residual subset R of symplectic diffeomorphisms such that if f belongs to R is not partially hyperbolic, then $h_{top}(f) = S(f)$.

Another important problem in smooth theory is about the regularity of the topological entropy. For instance, Katok proved that the topological entropy of a C^2 -diffeomorphism f is approximated by topological entropy of f restrict to hyperbolic sets, which allows to conclude lower semicontinuity of the entropy.^ă In the lack of partial āhyperbolicity we prove a similar result for C^1 -generic symplectic diffeomorphisms.^ă

Anosov Representations and Lorentzian Geometry

Thierry Barbot

Université d'Avignon

In this talk I will present the role of the concept of Anosov representations in the context of lorentzian geometry in constant curvature.

Periodic orbits in the restricted three body problem and Arnold's J^+ invariant.

Urs Frauenfelder

Universität Augsburg

This is joint work with Kai Cieliebak. After regularization periodic orbits in the restricted three body problem become knots in RP^3 . Their projection to position space is a curve in the plane. We show that Arnold's J^+ invariant is invariant under homotopies of periodic orbits and gives an additional invariant to their knot type in RP^3 .

Topological ergodicity and pointwise entropy

Vilton Pinheiro

UFBA

Let $f : M \rightarrow M$ be an expanding map defined on a Riemannian manifold M . It is well known the existence for a residual set $M' \subset M$ with historical behavior. That is, given $x \in M'$, the Birkhoff average $\frac{1}{n} \sum_{j=0}^{n-1} \varphi \circ f^j(x)$ does not converge for some continuous function φ . Although $\mu(M') = 0$ for every invariant probability μ , M' is a large set. Indeed, besides be residual, it has full Hausdorff dimension and also full topological entropy. We will use M' to introduce the concepts of topological ergodicity and pointwise entropy.

Symplectic embeddings, toric domains and billiards

Vinicius Gripp Barros Ramos

IMPA

Embedded contact homology capacities were defined by Michael Hutchings and they have been shown to provide sharp obstructions to many symplectic embeddings. In this talk, I will explain how they can be used to study symplectic embeddings of the lagrangian bidisk and how this space is related to the space of billiards on a round disk.

Rigidity of smooth critical circle maps

Welington de Melo

IMPA

I will discuss several rigidity results in dynamics including a recent result in collaboration with Pablo Guarino and Marco Martens where we prove that if two critical circle maps have the same irrational rotation number and the same criticality they conjugate by a C^1 diffeomorphism.

Sessão de Poster

Ciclos principais de uma distribuição de planos em \mathbb{R}^3

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Abstract

Neste trabalho vamos considerar uma distribuição de planos Δ_η definida por um campo de vetores $\eta : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, regular, e analisar o comportamento das folhas integrais compactas das folheações $\mathcal{F}_1(\eta)$ e $\mathcal{F}_2(\eta)$, definidas pelo sistema de equações diferenciais implícitas

$$2(D\eta(dr), dr, \eta) + \langle \text{rot}(\eta), \eta \rangle \cdot \langle dr, dr \rangle = 0, \quad \langle \eta, dr \rangle = 0,$$

que definem as linhas principais associadas as direções máximas e mínimas de um problema de multiplicador de Lagrange da função

$$k_\eta(dr) = -\frac{\langle D\eta(dr), dr \rangle}{\langle dr, dr \rangle},$$

que representa a curvatura normal do campo η , restrita a distribuição de planos Δ_η , distribuição essa completamente integrável ou não. Apresentaremos resultados de hiperbolideidade dessas folhas compactas que são denominadas de ciclos principais.

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Fractals and Hausdorff Dimension

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Abstract

We know, intuitively, from the Euclidean geometry that the dimension of a dot is 0, the dimension of a line is 1, the dimension of a square is 2 and the dimension of a cube is 3. However, there are other geometries, like the Fractal Geometry where we found objects with a fractional dimension. These objects are called fractals whose name comes from the verb *frangere*, in Latin, that means breaking, fragmenting.

In this work we will study about the concept of dimension, defining topological dimension and Hausdorff dimension. We will also present a definition of fractal using the Hausdorff dimension.

The purpose of this work, besides presenting the definitions of dimension, is to calculate the Hausdorff dimension of the Cantor set and verify that it is a fractal.

On the Anosov character of the Pappus-Schwartz representations

Viviane Pardini Valério - UFMG

Thierry Barbot - Université d'Avignon (advisor)

In the paper *Pappus's Theorem and The Modular Group* [2], R. E. Schwartz observed that the classical Pappus theorem gives rise to an action of the modular group on the space of marked boxes. He inferred from this a 2-dimensional family of faithful representations of the modular group into the group of projective symmetries. These representations, have a dynamical behavior very similar to the one of Anosov representations, even if they are never Anosov themselves. In this poster, we announce the main result of Viviane thesis [1] that elucidates this Anosov character of the Schwartz representations by proving that their restrictions to the index 2 subgroup are limits of Anosov representations.

[1] V. Pardini Valério, *Teorema de Pappus, Representações de Schwartz e Representações Anosov*. PhD thesis, UFMG - Brazil, January 2016,
<http://www.mat.ufmg.br/intranet-atual/pgmat/TesesDissertacoes/uploaded/Tese68.pdf>

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Approximation of Invariant Measures by Atomic Measures

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Abstract

Given a periodic point p of period n , the measure $\mu_p = \frac{1}{n}(\delta_p + \delta_{f(p)} + \cdots + \delta_{f^{n-1}(p)})$ is called an atomic measure. This measure is invariant under f and it is ergodic.

An interesting result is that each invariant measure can be approximated by atomic measures. Formally, given an expanding map $f: M \rightarrow M$, M a compact set, every invariant probability μ under f can be approximated by invariant probabilities supported on periodic orbits (atomic measures) in the weak-* topology,

From both theoretical and computational point of view, atomic measures provide great convenience. For example: let f be an expanding map and let ν be a measure supported on the periodic orbit of p , p a periodic point of period N (that is, $\nu = \frac{1}{N}(\delta_p + \delta_{f(p)} + \cdots + \delta_{f^{N-1}(p)})$). Then, given an integrable function φ , we have that:

$$\int \varphi d\nu = \frac{1}{N} \sum_{i=0}^{N-1} \varphi(f^i(x))$$

for every $x \in \{p, f(p), \dots, f^{N-1}(p)\}$.

We will provide a complete proof of this important result, based on the specification property of expanding maps on compact sets.

DISCRETE SYNCHRONIZATION OF MASSIVELY CONNECTED SYSTEMS USING HIERARCHICAL COUPLINGS

CAMILLE POIGNARD

We study the synchronization of massively connected dynamical systems for which the interactions come from the succession of couplings forming a global hierarchical coupling process. Motivations of this work come from the growing necessity of understanding properties of complex systems that often exhibit a hierarchical structure. Starting with a set of 2^n systems, the couplings we consider represent a two-by-two matching process that gather them in greater and greater groups of systems, providing to the whole set a structure in n stages, corresponding to n scales of hierarchy. This leads us naturally to the synchronization of a Cantor set of systems, indexed by $\{0, 1\}^{\mathbb{N}}$, using the closed-open sets defined by n -tuples of 0 and 1 that permit us to make the link with the finite previous situation of 2^n systems: we obtain a global synchronization result generalizing this case. In the same context, we deal with this question when some defects appear in the hierarchy, that is to say when some couplings among certain systems do not happen at a given stage of the hierarchy. We prove we can accept an infinite number of broken links inside the hierarchy while keeping a local synchronization, under the condition that these defects are present at the N smallest scales of the hierarchy (for a fixed integer N) and they be enough spaced out in those scales.

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Uniqueness of SRB measures for Endomorphism

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ABSTRACT

Let f be a C^2 Endomorphism (local diffeomorphism), of a closed Riemannian manifold M without zero Lyapunov exponents and with k fixed index ($\dim E^s = k$). In [4] it was shown that the number of ergodic hyperbolic measures of f with SRB property is less than equal to the maximal cardinality of k -eskeleton (hyperbolic periodic points without any homoclinic relation[5]). The natural question appears in continuation of this work, regarding to [1], is a condition which implies the uniqueness of measures with SRB property.

In 1994 I.Kan [2] discovers some un-expecting example with intermingled basins which in case of two-cylinder are shown to be robustly transitive but without a unique SRB measure.[3] Defining the notion of "pre-transitivity" for endomorphisms, we see that the existence of a hyperbolic pre- transitive periodic point, can imply the uniqueness of SRB measures which in case, do not appears in Kan-type examples. we see that as a consequence it drives the ergodicity of the system in a conservative setting.

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DYNAMICS OF PIECEWISE SMOOTH SYSTEMS NEAR A TYPICAL CYCLE

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In this talk we concern with a class of dynamical systems obtained by general perturbations autonomous of an autonomous piecewise smooth system that presents a typical cycle. It is shown that under certain generic conditions on the unperturbed problem branch off into a certain number of periodic orbits for the perturbed problem.

Co-autores: D.D. Novaes (IMECC/UNICAMP), T.M. Seara (UPC/Barcelona) and M.A. Teixeira (IMECC/UNICAMP).

DA diffeomorphisms with pathological non compact two dimensional center foliation

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Abstract

In this poster we will present an open set of volume preserving derived from Anosov diffeomorphisms of the torus \mathbb{T}^4 with non absolutely continuous two dimensional center foliation by non compact leaves. More generally, let $f : \mathbb{T}^m \rightarrow \mathbb{T}^m$ be a DA diffeomorphism with linearization A , satisfying:

1. $\dim E_f^c = \dim E_A^c = d \geq 2$;
2. $E_A^c = E_1^c \oplus E_2^c \oplus \cdots \oplus E_d^c$ and $\lambda_i^c(A) > 0$;
3. f is dynamically coherent and in the universal cover $\angle(T_x \widetilde{W}_f^c, (\widetilde{E}_A^c)^\perp) > \alpha > 0$ for all $x \in \mathbb{R}^m$.

If the center foliation is absolutely continuous, then the center Lyapunov exponents satisfy $\sum_i \lambda_i^c(f, x) \leq \sum_i \lambda_i^c(A)$ for m almost everywhere $x \in \mathbb{T}^m$.

Also we find an open set $\mathcal{U} \subset PH_m^r(\mathbb{T}^4)$, $r \geq 2$, of DA diffeomorphisms far from Anosov diffeomorphisms such that each $f \in \mathcal{U}$ satisfies the conditions above, but $\sum_i \lambda_i^c(f, x) > \sum_i \lambda_i^c(A)$ for m almost everywhere $x \in \mathbb{T}^m$. Using the result above the center foliation W_f^c is non absolutely continuous with non compact leaves for each $f \in \mathcal{U}$.

Comparação dos comportamentos de um oscilador não linear com um AMS de acordo com a modelagem das vibrações

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Abstract

Amortecedor de Massaa Sintonizada (AMS) são dispositivos de controle passivo de vibrações bem consolidados em várias áreas como a construção civil, sendo aplicado em arranha céus, torres de transmissão, pontes, entre outros. Por simplificação, o projeto desses dispositivos consideram-se excitações periódicas, de frequência bem definida e independentes do movimento do sistema. Porém, em alguns casos, essa modelagem não é válida, pois a excitação varia em amplitude, frequência e potência. Nesses casos as fontes não ideais se apresentam como uma nova proposta para modelar excitações de baixa potência e que são influenciadas pela resposta do sistema excitado.

Desse modo, neste trabalho, são comparadas duas formas de modelagem das excitações aplicadas em um Oscilador Não Linear (ONL) ao qual é acoplado a um AMS. As equações diferenciais ordinárias que descrevem o problema com as diferentes excitações foram obtidas, assim como suas matrizes Jacobianas. Desenvolveu-se um algoritmo computacional que calculou os autovalores da matriz para uma faixa de valores dos parâmetros de rigidez e os classificou segundo o critério de Lyapunov. Ou seja, verificou-se o comportamento do sistema (estável/instável) qualitativamente. Construiu-se, então, planos de fases para visualizações quantitativas do comportamento do oscilador não linear considerando parâmetros e condições iniciais iguais para o sistema com cada uma duas formas de excitação.

O sistema como um todo apresentou-se estável para todos os parâmetros testados, quando excitado pela força periódica. Contudo, quando acoplada a fonte não ideal, ele é instável em todos os casos. Verificando-se os planos de fases para um caso particular dos parâmetros, observa-se grande aumento da amplitude máxima apesar do AMS absorver parte das vibrações.

Linearização Equivalente para Análise de Folga em Superfícies de Controle de Aeronaves

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Abstract

Sistemas mecânicos em movimento em meio fluido podem experimentar diversos fenômenos físicos. Entre tais fenômenos alguns recebem maior atenção de engenheiros e projetistas durante os ciclos de desenvolvimento de produtos. Em particular, as oscilações de ciclo limite (OCLs) em superfícies de controle de aeronaves são potencialmente destrutivas, principalmente pela capacidade de redução de ciclo de vida por fadiga, além dos prejuízos à aeronavegabilidade. Assim, OCLs podem comprometer a integridade estrutural de aviões e, consequentemente, a segurança dos passageiros e tripulação.

A dinâmica de uma OCL tem característica principal elementos não lineares nos modelos matemáticos que descrevem o movimento. Para uma superfície com folga de amplitude δ , descrita por um modelo de um grau de liberdade (gdl), tem-se uma equação diferencial do tipo $m\ddot{u}(t) + k_{nl}u(t) = 0$, sendo k_{nl} a rigidez não linear $u(t)$ o deslocamento e m sua massa. A força elástica restauradora $f_{k_{nl}}(t) = k_{nl}u(t)$ é nula se $|u(t)| \leq \delta$ ou $f_{k_{nl}}(t) = k[u(t) - \delta]$ se $|u(t)| \geq \delta$, para k uma rigidez linear constante.

O sistema acima descrito pode ser analisado usando uma representação idealizada de um sistema linear equivalente. Tal representação permite se obter uma expressão envolvendo a amplitude de folga δ , a rigidez nominal k , uma amplitude de oscilação A e uma rigidez equivalente k_{eq} do sistema idealizado. Esta abordagem viabiliza o estudo das OCLs, incluindo a predição de amplitudes de oscilações em problemas de interação entre fluido e estrutura, característicos destas análises de aviões com folga em superfície de controle. Nesta referida formulação assume-se um movimento harmônico com $u(t) = A \sin(\omega t)$, sendo ω a frequência da OCL e t o tempo.

A estratégia apresentada pode ser executada pela solução analítica da equação do movimento não linear. No entanto, não é a mais adequada para assimetria de amplitude de folga nem mesmo de rigidez. Então, para estes casos, pode-se utilizar o método de Balanço Harmônico que, em particular para movimentos com um harmônico fundamental, é comumente denominado método de *Linearização Equivalente* (MLE). O MLE também permite se obter a relação entre os parâmetros físicos do movimento, denominada *função descritiva*, adequada para se encontrar a solução do problema de OCL.

Contudo, o presente trabalho apresenta uma comparação entre a solução analítica e o MLE para solução do problema de OCL. O estudo de caso apresentado para um sistema de um gdl pode ser generalizado para soluções envolvendo modelos dinâmicos de aeronaves e suas superfícies de controle. Portanto, trata-se de um tema de significativa relevância, especialmente para a indústria aeronáutica.

ISOCRONICIDADE PARA SISTEMAS HAMILTONIANOS PLANARES POLINOMIAIS DE GRAUS 5 E 7

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Neste trabalho caracterizamos completamente os centros isócronos Hamiltonianos polinomiais triviais de graus 5 e 7. Precisamente, fornecemos formulas simples, com mudanças de coordenadas lineares, para Hamiltonianos da forma $H = (f_1^2 + f_2^2)/2$, onde $f = (f_1, f_2) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ e uma função polinomial com $\det Df = 1$, $f(0,0) = (0,0)$ e o grau de f é 3 ou 4.

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Co-autor: Francisco Braun, *Universidade de São Carlos* e Jaume Llibre, Universitat Autònoma de Barcelona.

Comparação dos comportamentos de um oscilador não linear com um AMS de acordo com a modelagem das vibrações

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Abstract

Amortecedor de Massaa Sintonizada (AMS) são dispositivos de controle passivo de vibrações bem consolidados em áreas como a construção civil, sendo aplicado em arranha céus, torres de transmissão, pontes, entre outros. Por simplificação, para o projeto desses dispositivos considera-se excitações periódicas e de frequência bem definida e independentes do movimento do sistema. Porém, em alguns casos, essa modelagem não válida, pois a excitação varia em amplitude, frequência e potência. Nesses casos, fontes não ideais se apresentam como uma nova proposta para modelar excitações de baixa potência e que são influenciadas pela resposta do sistema excitado.

Desse modo, neste trabalho, são comparadas as duas formas de modelagem aplicadas em um Oscilador Não Linear (ONL) ao qual é acoplado a um AMS. As equações que descrevem o problema com as diferentes excitações foram obtidas, assim como suas matrizes Jacobianas. Desenvolveu-se um algoritmo computacional que calculou os autovalores da matriz para uma faixa de valores dos parâmetros de rigidez e os classificou segundo o critério de Lyapunov. Ou seja, verificou-se o comportamento do sistema (estável/instável) qualitativamente. Construiu-se, então, planos de fases para vizualização quantitativa do comportamento do oscilador não linear considerando parâmetros e condições iniciais iguais para o sistema com cada uma duas formas de excitação.

O sistema como um todo apresentou-se estável para todos os parâmetros testados, quando excitado pela força periódica. Contudo, quando acoplada a fonte não ideal, ele é instável em todos os casos. Comparando através do plano de fases para um caso particular dos parâmetros, vemos grande aumento da amplitude máxima apesar do AMS absorver parte das vibrações.

On Structural Stability of 3D Piecewise Smooth Vector Fields

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Abstract

Nowadays there exists an extensive range of phenomena which present discontinuous motion and, thus, the theory of nonsmooth dynamical systems (or Filippov systems) has been a common topic of interest in many fields as Mathematics, Physics, Engineering, Geophysics and correlated areas.

In this context, the structural stability of a NSDS is a very active research topic due to the interest to know the efficiency of a model with respect to the initial conditions and parameters. Nevertheless, in higher dimension, a NSDS may present a very rich and complex dynamics, which makes it very difficult to classify the structurally stable systems. In the literature, most works deal with local structural stability and there is no much about global phenomena around the switching manifold.

Let M be an oriented compact 3-manifold and Σ be an embedded codimension one submanifold of M which is connected and simply connected. Denote the set of nonsmooth vector fields Z on M with switching manifold Σ by $\Omega^r(M)$.

In this work, aspects of the structural stability in $\Omega^r(M)$ are studied. We characterize the set Ω_Σ^r constituted by the elements $Z \in \Omega^r(M)$ which are structurally stable on Σ . In addition, we show that this set is residual in $\Omega^r(M)$. We also characterize a class of elements $Z \in \Omega^r(M)$ which are structurally stable around Σ in M .

Quasisymmetric rigidity of multicritical circle maps

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Abstract

We discuss the rigidity problem for multicritical circle maps, that is, smooth homeomorphisms of the circle having a finite number of critical points, all of them being of the non-flat type.

It is known from Yoccoz, that any two multicritical circle maps with the same irrational rotation number are topologically conjugated. The question then arises of the smoothness of this conjugacy. It has been “almost” completely answered in the case of only one critical point. However, we cannot say much about it in the multicritical case.

In this work, we present a preliminary step in this direction proving that the conjugacy that sends critical points to critical points is quasi-symmetric. Joint work with Edson de Faria.

Shil'nikov phenomenon in discontinuous piecewise linear differential systems in \mathbb{R}^3

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Abstract

In 1965 Shilnikov ensured under assumptions on the eigenvalues of its linearization, the existence of infinitely many unstable periodics orbits in every neighborhood of a homoclinic orbit associated to a saddle-focus equilibrium point for smooth differential systems in \mathbb{R}^3 . Later on in 2007 Llibre, Ponce and Teruel proved that this Shil'nikov dynamics also exists for continuous piecewise linear differential systems studied numerically previously by Arneodo, Coullet and Tresser in 1981. Our goal is to show that the Shil'nikov dynamics also is exhibited in discontinuous piecewise linear differential systems.

This work is made in collaboration with Jaume Llibre and Ricardo Miranda Martins.

Reações químicas e equações de Linenard generalizadas

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Abstract

Ciclos limites são procurados em um modelo matemático de uma reação química hipotética que envolve essencialmente duas espécies reagentes. Fisicamente, estes ciclos limites correspondem às oscilações de tempo periódico nas concentrações dos dois produtos químicos. Um modelo que exibe oscilações periódicas é o autocatalisador cúbico, proposto por Gray and Scott [1]. Métodos numéricos revelaram, na década de 90, que o comportamento de ciclo limite nesse modelo somente é possível em uma região restrita do espaço de parâmetros. Evidência numérica foi apresentada para afirmar que o ciclo limite é único e estável à perturbações infinitesimais. Em 2005, Hwang e Tsai [2] provaram a unicidade dos ciclos limites estáveis usando equações de Lienard generalizadas.

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Fractais e Dimensão de Hausdorff

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Abstract

Da geometria euclidiana sabemos, intuitivamente, que um ponto tem dimensão 0, uma reta tem dimensão 1, um plano tem dimensão 2 e um cubo tem dimensão 3. Porém, existem outras geometrias, como a geometria fractal na qual encontramos objetos matemáticos que possuem dimensão fracionária. Esses objetos são denominados fractais cujo nome vem do verbo *frangere*, em latim, que significa quebrar, fragmentar.

Neste trabalho faremos um estudo sobre o conceito de dimensão, definindo dimensão topológica e dimensão de Hausdorff. Apresentaremos também uma definição de fractal utilizando a dimensão de Hausdorff.

O objetivo deste trabalho é, além de apresentar as definições de dimensão, também calcular a dimensão de Hausdorff do Conjunto de Cantor e verificar que o mesmo é um fractal.

Teorema Twist de Moser e o Modelo May

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Resumo

Apresentaremos uma aplicação do Teorema Twist de Moser à dinâmica populacional do Modelo May seguindo a exposição feita em [1]. O Modelo May descreve a evolução discreta de um sistema de parasitas e hospedeiros dada por

$$\begin{aligned}x_{n+1} &= ax_n f(x_n, y_n) \\y_{n+1} &= cx_n(1 - f(x_n, y_n))\end{aligned}$$

Onde x_n e y_n são a população de hospedeiros e de parasitas, respectivamente, na n -ésima geração, e a função f dada por $f(x_n, y_n) = [(1 + by_n)/k]^{-k}$ representa a fração de hospedeiros que não estão infectados e k é um parâmetro real. Tal modelo descreve, por exemplo, a dinâmica populacional entre o parasita *Pleolophus basizonus* e o hospedeiro *Neodiprion sertifer*, e sob uma certa mudança de coordenadas preserva área e orientação. Além disso possui um ponto elíptico não degenerado e com isso podemos aplicar o Teorema Twist de Moser e assim obter uma família de curvas invariantes perto de tal ponto. O Modelo May descreve

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Sobre a não existência, existência e unicidade de ciclos limites para campos vetoriais planares

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Abstract

Neste trabalho, estudamos ciclos limites de campos de vetores planares. O objetivo foi apresentar critérios sobre a não existência, existência e unicidade desses ciclos limites. Descreveremos a seguir dois desses critérios estudados.

Primeiro Critério: Sejam (P, Q) um campo de vetores C^1 definido em um aberto U do \mathbb{R}^2 , $(u(t), v(t))$ uma solução periódica de (P, Q) de período T , $R : U \rightarrow \mathbb{R}$ uma aplicação C^1 tal que $\int_0^T R(u(t), v(t))dt \neq 0$ e $V = V(x, y)$ uma solução C^1 da seguinte equação

$$P \frac{\partial V}{\partial x} + Q \frac{\partial V}{\partial y} = RV, \quad (x, y) \in U.$$

Então, a trajetória fechada $\gamma = \{(u(t), v(t)) \in U; t \in [0, T]\}$ está contida em $\Sigma = \{(x, y) \in U; V(x, y) = 0\}$, e γ não está contida em um anel periódico do campo (P, Q) . Além disso, se o campo de vetores (P, Q) é analítico, então γ é um ciclo limite.

Segundo Critério: Sejam (P, Q) um campo de vetores C^1 definido em um aberto U do \mathbb{R}^2 , $V = V(x, y)$ uma solução C^1 da equação diferencial parcial linear

$$P \frac{\partial V}{\partial x} + Q \frac{\partial V}{\partial y} = \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) V.$$

Se γ é um ciclo limite do campo (P, Q) , então γ está contida em $\Sigma = \{(x, y) \in U; V(x, y) = 0\}$.

Como aplicação, utilizaremos esses dois critérios em algumas famílias de campos de vetores polinomiais quadráticos e cúbicos para calcular o número de ciclos limites que bifurcam do centro linear

$$\dot{x} = -y, \quad \dot{y} = x,$$

quando o perturbamos da forma

$$\dot{x} = -y + \varepsilon \sum_{i+j=1}^n a_{ij} x^i y^j, \quad \dot{y} = x + \varepsilon \sum_{i+j=1}^n b_{ij} x^i y^j.$$

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Modelo predador-presa com estratégia de colheita com limiar contínuo

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Abstract

Atualmente muitos trabalhos têm sido desenvolvidos para analisar modelos predador-presa com limiar de colheita impulsivo. Os autores investigam modelos onde uma estratégia de controle de realimentao com limiar impulsivo é tomada somente quando a densidade das presas atinge um limite. Por outro lado, outros autores formularam uma forma alternativa de colheita com limiar contínuo e investigaram suas propriedades no âmbito de um modelo predador-presa. A presença de um limiar de colheita é interessante e faz o comportamento dinâmico mais complexo. Portanto, é necessário considerar como o limiar afeta a dinâmica do ecossistema. Collie e Spencer, em um de seus trabalhos, analisam a dinâmica de um modelo predador-presa considerando as estratégias de colheita com limiar contínuo de forma que quando a população predador está acima de um certo limite, a estratégia de colheita ocorre e quando esta densidade está abaixo do limite, as estratégias são encerradas. Uma análise adicional tem sido desenvolvida desde então. Um modelo predador-presa com essa política é descrito pelo seguinte sistema

$$(\dot{x}, \dot{y}) = \left(rx \left(1 - \frac{x}{K} \right) - \frac{\beta xy}{\alpha + x}, \frac{\beta_1 xy}{\alpha + x} - \delta y - H(y) \right), \quad (1)$$

onde os parâmetros r é a taxa intrínseca de crescimento da população presa, K é a capacidade de suporte para as presas, δ é a taxa de morte do predador, β é a taxa máxima de captação por predador, β_1 denota a proporção de conversão de biomassa com $0 < \beta_1 < \beta$, α é a constante de meia saturação para uma função resposta Holling tipo II, α contribui para o crescimento do predador, e a função colheita $H(y)$ significa que a colheita inicia quando a densidade de predadores ultrapassa o limite T , e decresce suavemente para o valor limite de saturação h . Além disso, $H(y)$ é definida por

$$H(y) = \begin{cases} 0, & \text{se } y \leq T, \\ \frac{h(y-T)}{c+y-T}, & \text{se } y > T. \end{cases} \quad (2)$$

Observe que os parâmetros definidos acima são todos positivos, e a segunda coordenada do sistema é sempre negativa se $\beta_1 < \delta$, assim assumimos $\beta_1 - \delta > 0$.

O objetivo desse trabalho é investigar o comportamento qualitativo global do modelo predador-presa quanto à estabilidade do sistema e tipos de bifurcações que ocorrem.

On the Lyapunov spectrum of relative transfer operators

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Abstract

We analyze the Lyapunov spectrum of the relative Ruelle operator associated with a skew product whose base is an ergodic automorphism and whose fibers are full shifts. We prove that these operators can be approximated in the C^0 -topology by positive matrices with an associated dominated splitting. The underlying arguments are two-fold: the dominated splittings are obtained through an application of a theorem of Bochi-Viana on accessible cocycles, whereas the approximation in the C^0 -topology lead to a new type of convergence theorems for quotients of transfer operators, which is of independent interest. Joint work with Mário Bessa.

EXPANSIVE FLOWS AND THEIR CENTRALIZERS

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Abstract

In this we report on [?], which is part of my PhD Thesis. In it we study the centralizer of flows and \mathbb{R}^d -actions on compact Riemannian manifolds. We prove that the centralizer of every C^∞ Komuro-expansive flow with non-resonant singularities is trivial, meaning it is the smallest possible, and deduce there exists an open and dense subset of geometric Lorenz attractors with trivial centralizer. We show that \mathbb{R}^d -actions obtained as suspension of \mathbb{Z}^d -actions are expansive if and only if the same holds for the \mathbb{Z}^d -actions. We also show that homogeneous expansive \mathbb{R}^d -actions have quasi-trivial centralizers, meaning that it consists of orbit invariant, continuous linear reparametrizations of the \mathbb{R}^d -action. In particular, homogeneous Anosov \mathbb{R}^d -actions have quasi-trivial centralizer.

Um estudo sobre a família quadrática

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Abstract

The beginning of the studies in Dynamical Systems starts with the Differential and Integral Calculus discovered by Newton and Leibniz, in order to solve problems caused by physical and geometrical considerations. These methods gradually conducted the consolidation of Differential Equations as a new branch of mathematics, which in the mid-eighteenth century became one of the most important disciplines and the most effective method for scientific research. In some of these systems many complicated behaviors are observed using equations. An algebraic form does not indicate that the dynamic behavior of this system is simple, it can even get to be "chaotic". In this work we intend to study the behavior of supposedly simple dynamical systems, in the form of the quadratic family $F_\mu(x) = \mu x(1 - x)$.

Key-Words: Differential Equations, Dynamical Systems, Quadratic Family.

The Gurevic entropy for Markov shifts

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Abstract

Let \mathcal{A} be an alphabet, the full \mathcal{A} -shift is the collection of all bi-infinite sequences with symbols from \mathcal{A} . The full \mathcal{A} -shift is denoted by

$$\mathcal{A}^{\mathbb{Z}} = \{(x_i)_{i \in \mathbb{Z}} : x_i \in \mathcal{A}, \text{ for all } i \in \mathbb{Z}\}$$

The shift map on the full shift is a map $\sigma : \mathcal{A}^{\mathbb{Z}} \rightarrow \mathcal{A}^{\mathbb{Z}}$ that associates a point $x \in \mathcal{A}^{\mathbb{Z}}$ to the point whose i^{th} coordinate is $\sigma(x_i) = x_{i+1}$.

A subset $\mathcal{S} \subseteq \mathcal{A}^{\mathbb{Z}}$ is called a subshift over \mathcal{A} if \mathcal{S} is closed with respect to the topology of $\mathcal{A}^{\mathbb{Z}}$ and if \mathcal{S} is invariant under the shift map, that is, $\sigma(\mathcal{S}) \subseteq \mathcal{S}$.

A Markov shift is a subshift which can be associated to a set \mathcal{S}_G of bi-infinite walks on the edges of a countable directed graph G . The Markov shift \mathcal{S} is locally compact if and only if G has finite in- and out-degree. The Markov shift is compact if and only if G is a finite graph if and only if it is a shift finite.

For a Markov shift \mathcal{S} , let $Per(\mathcal{S})$ be the set of periodic points of \mathcal{S} , that is, $Per(\mathcal{S}) = \{y \in \mathcal{S} : \sigma^n(y) = y \text{ for any } n \in \mathbb{Z}\}$.

Let \mathcal{S} and \mathcal{T} be subshifts, a factor map $f : \mathcal{S} \rightarrow \mathcal{T}$ is a continuous shift commuting onto map. We say that a fiber of f on $y \in \mathcal{T}$ is the preimage set $f^{-1}(y)$. Moreover, f is said to be bounded-to-1 if there is some $M \in \mathbb{N}$ such that all fibers of f have cardinality at most M ; f is finite-to-1 if all fibers are finite sets; and f is countable-to-1 if all fibers are countable sets.

The 1-point compactification \mathcal{S}_0 of a locally compact subshift \mathcal{S} is the compact metric dynamical system which consists of the Alexandroff 1-point compactification of the shift space with the extended shift maps.

The Gurevic entropy is defined to be the topological entropy of the 1-point compactification of the subshift.

$$h_G(\mathcal{S}) = h_{top}(\mathcal{S}_0)$$

We consider the Gurevic metric the metric on \mathcal{S} such that the completion of \mathcal{S} with respect to this metric is \mathcal{S}_0 .

Let \mathcal{S} and \mathcal{T} be transitive locally compact Markov shifts. A factor map $f : \mathcal{S} \rightarrow \mathcal{T}$ is proper if $f^{-1}(K)$ is a compact set for every compact set $K \subseteq \mathcal{T}$.

In this work, we present relationships between the Gurevic entropies of two transitive locally compact Markov shifts under some conditions on the factor maps between them.

Theorem: Let \mathcal{S} be a transitive locally compact Markov shift and \mathcal{T} a subshift locally compact. Let $f : \mathcal{S} \rightarrow \mathcal{T}$ a factor map. If the fiber $f^{-1}(y)$ is countable for every $y \in Per(\mathcal{T})$ then

$$h_G(\mathcal{S}) \leq h_G(\mathcal{T}).$$

In particular, if f is countable-to-1, so $h_G(\mathcal{S}) \leq h_G(\mathcal{T})$.

Theorem: Let \mathcal{S}, \mathcal{T} be transitive locally compact Markov shifts and let $f : \mathcal{S} \rightarrow \mathcal{T}$ a factor map finite-to-1. Then

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Theorem: Let \mathcal{S}, \mathcal{T} be locally compact Markov subshifts and let $f : \mathcal{S} \rightarrow \mathcal{T}$ a factor map proper. Then

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The Gurevic entropy for Markov shifts

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$$h_G(\mathcal{S}) = h_G(\mathcal{T}).$$

Theorem: Let \mathcal{S}, \mathcal{T} be locally compact Markov subshifts and let $f : \mathcal{S} \rightarrow \mathcal{T}$ a factor map proper. Then

$$h_G(\mathcal{S}) \geq h_G(\mathcal{T}).$$

Theorem: Let \mathcal{S}, \mathcal{T} be locally compact Markov subshifts and let $f : \mathcal{S} \rightarrow \mathcal{T}$ a factor map countable-to-1 proper. Then

$$h_G(\mathcal{S}) = h_G(\mathcal{T}).$$

Fractals and Hausdorff Dimension

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Abstract

We know, intuitively, from the Euclidean geometry that the dimension of a dot is 0, the dimension of a line is 1, the dimension of a square is 2 and the dimension of a cube is 3. However, there are other geometries, like the Fractal Geometry where we found objects with a fractional dimension. These objects are called fractals whose name comes from the verb *frangere*, in Latin, that means breaking, fragmenting.

In this work we will study about the concept of dimension, defining topological dimension and Hausdorff dimension. We will also present a definition of fractal using the Hausdorff dimension.

The purpose of this work, besides presenting the definitions of dimension, is to calculate the Hausdorff dimension of the Cantor set and verify that it is a fractal.

On a degenerate cycle through a cusp-regular point

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Abstract

The study of nonsmooth vector fields has been developed very fast in recent years and it has become a common frontier between Mathematics, Physics and Engineering. It can be considered a recent research field, so there are plenty of open questions. Nonsmooth systems admit lots of typical cycles that do not occur in smooth systems, therefore the study of bifurcations of these cycles becomes a interesting topic of research.

In this work we just consider planar discontinuous vector fields having a straight line as the set of discontinuity. Our main objective is to study the qualitative behavior of a typical cycle through a cusp-regular singularity. The approach is divided in two parts: the first one is to analyze the local bifurcation of the singularity and the second one is to study the first return map defined near the cycle. In addition, the bifurcation diagram for a model presenting this kind of cycle is presented.

Bifurcações Genéricas e Relações de Equivalência em Campos de Vetores Suaves por Partes

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Abstract

Neste trabalho abordaremos aspectos qualitativos e geométricos de Campos de Vetores Suaves por Partes (CVSP's ou Campos de Filippov). Consideraremos $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ uma função suave que tem $0 \in \mathbb{R}$ como valor regular. Definimos $\Sigma = f^{-1}(0)$, de tal forma que dividiremos o plano \mathbb{R}^2 em duas regiões: $\Sigma^+ = \{p \in \mathbb{R}^2 / f(p) > 0\}$ e $\Sigma^- = \{p \in \mathbb{R}^2 / f(p) < 0\}$. Nas regiões Σ^+ e Σ^- atuarão os campos contínuos X e Y respectivamente, ambos de classe C^r . Assim, um Campo de Filippov $Z : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ é definido por

$$Z(p) = \begin{cases} X(p), & \text{se } p \in \Sigma^+, \\ Y(p), & \text{se } p \in \Sigma^-. \end{cases}$$

Serão introduzidos os conceitos de equivalência topológica e Σ -equivalência entre Campos de Filippov, bem como os conceitos de estabilidade e Σ -estabilidade estrutural. A partir disso, iremos determinar as formas normais de CVSP's definidos numa vizinhança de pontos regulares ou de singularidades genéricas.

Por fim, classificaremos e caracterizaremos bifurcações genéricas locais e globais de codimensão 1 por meio do retrato de fase e do diagrama de bifurcação dos campos envolvidos, que é o objetivo principal deste trabalho. Este estudo será feito tanto desdobrando CVSP's cuja representação envolve apenas um parâmetro, quanto para CVSP's cuja forma normal apresenta dois parâmetros. Em geral, nos CVSP's estudados aparecerão pontos de equilíbrio sobre Σ ou tangências das trajetórias dos campos com Σ ou até mesmo a simultaneidade destes fenômenos.

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Limit Cycles in Continuous Piecewise Linear Systems

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Abstract

In this poster we present some results on limit cycles of continuous piecewise linear systems with three regions. In particular, we present a method to study the existence of limit cycles, a method for the study of the stability of these cycles, and two applications of these methods.

Generic Properties of Magnetic Flows

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Abstract

The perturbative theory on dynamical systems is one of the most powerful ones to describe robust and generic properties of dynamical systems. Previous perturbations theorems were Kupka-Smale's theorem and the Franks' lemma. Those perturbations tools were generalized to more difficult settings: vector fields, Hamiltonians, etc. However, there are very important dynamics coming from the differential geometry: geodesic flows. The understanding of this flow quickly by Hopf and Anosov, that give us tools for the understanding of the dynamics and statistics of that flow for negatively curved Riemannian manifolds.

The first ones to deal with Kupka-Smale's theorem for geodesic flows were Klingenberg, Takens and Anosov. They were delicate computations and do not were suitable to obtain a Franks-type result. It was Contreras and Paternain, the first ones to obtain Franks' lemma for geodesic flows, but for surfaces. After many years, Contreras was able to obtain Franks' lemma for any dimension (geodesic flows).

The geodesic equation can be view like the Hamiltonian flow of the Kinetic Energy. Another well known flows, induced from some physical phenomena, are the magnetic flows can be written as the following equation,

$$\begin{cases} \dot{x} = v, \\ \nabla_v v = Y_x(v), \end{cases}$$

where Y is the so called Lorentz force. The magnetic flow was studied by Miranda but only in dimension two [2, 3].

Our main results is to give a full proof of Kupka-Smale's theorem and Franks' lemma for magnetic flows in any dimension.

Our method differs to the previous ones, since we use new results from the Control Theory that was used by Rifford and Ruggiero [4], to obtain similar results for Hamiltonians perturbations.

Let us explain the differences. There are three objects in the analysis: the Hamiltonian (that is the kinetic force in the geodesic and magnetic flow), the symplectic form, and the metric. For geodesic flows, the metric is perturbed. In Rifford and Ruggiero article [4, 1], the Hamiltonian is perturbed. In our work the symplectic form is perturbed. To obtain these results, one of the key arguments is to obtain nice coordinates to transform our analysis in a equation to apply the control theory.

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On the integrability of a three-dimensional forced-damped differential system

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Abstract

In 2011 Pehlivan proposed a three-dimensional forced-damped autonomous differential system which can display simultaneously unbounded and chaotic solutions. This untypical phenomenon has been studied recently by several authors. In this poster we study the opposite to its chaotic motion, i.e. its integrability, mainly the existence of polynomial and rational first integrals through the analysis of its invariant algebraic surfaces.

Stability of non-deterministic systems

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Abstract

We define non-deterministic dynamic systems in a compact metric space. In the space of non-deterministic dynamics we introduced some semigroups. The compactness of metric space cited assure the existence of finitely many invariant classes and their periodicity. We characterize the combinatorial stability and prove abundance of combinatorially stable dynamic systems: these systems form an open and dense family among semigroups of non-deterministic transformations.

Expanding structures

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Abstract

In this work we prove we consider families of transformations in multidimensional Riemannian manifolds with non-uniformly expanding behavior. We give sufficient conditions for the continuous variation, in the weak-* topology, of expanding measures for those transformations. We also conclude that these conditions can be stated in terms of the positiveness of the Lyapunov exponents and obtain that the first hyperbolic time map also varies continuously with the dynamics. In particular we conclude that the set of expanding measures for a fixed dynamics is compact in the weak-* topology.

C^r -stabilisation of non-transverse heterodimensional cycles

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Abstract

A diffeomorphism f has a *heterodimensional cycle* if there are hyperbolic sets with different indices whose invariant sets meet cyclically. The cycle is *C^r -robust* if diffeomorphisms g C^r -close to f have a cycle associated to the continuations of the hyperbolic sets. When the cycle is associated to a pair of saddles the cycle is *stabilised* if there are nearby diffeomorphisms with a robust cycle associated to hyperbolic sets containing the continuations of these saddles.

In dimension three, we prove the C^r -stabilisation, $r \geq 2$, of a class of heterodimensional cycles with heterodimensional tangencies (the two dimensional invariant manifolds of the saddles have a quadratic intersection).

A key ingredient of our method is a renormalisation scheme at the heteroclinic quadratic intersection converging to a Hénon-like family of endomorphism with blenders.

Dinâmica do Círculo: O Teorema de Denjoy

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Resumo

Este trabalho visa o estudo do Teorema de Denjoy, mas para isso apresentaremos alguns conceitos importantes da dinâmica topológica e da dinâmica do círculo que serão necessários para o estudo desse Teorema. Inicialmente, abordaremos algumas definições importantes para o estudo da dinâmica topológica como as seguintes:

Definição 1 Seja $f : M \rightarrow M$ um sistema dinâmico discreto e $x \in M$, definimos o ω -limite de x como: $\omega(x, f) = \{y \in M : \exists n_k \rightarrow +\infty; f^{n_k} \rightarrow y\}$.

Definição 2 Sejam $f : S^1 \rightarrow S^1$ e $x \in S^1$. Dizemos que x é um ponto recorrente se $\forall V$ vizinhança de $x \exists n \geq 1$ tal que $f^n(x) \in V$.

Dinâmica de uma Rotação: $R_\alpha(x) = x + \alpha \pmod{1}$.

Agora definiremos conceitos para o estudo da dinâmica do círculo.

Definição 3 Seja f um homeomorfismo do S^1 . Dizemos que f é conjugado a rotação de ângulo α se existir um homeomorfismo $h : S^1 \rightarrow S^1$ tal que $f \circ h = h \circ R_\alpha$. Ou, equivalente mente, $f = h \circ R_\alpha \circ h^{-1}$.

Definição 4 Se $F : \mathbb{R} \rightarrow \mathbb{R}$ contínua que verifica $\pi \circ F = f \circ \pi$, então dizemos que F é levantamento de f .

Definição 5 Seja F um levantamento de $f \in Hom_+(S^1)$. Definimos o número de rotação de um levantamento F , como $\rho(F) = \lim_{n \rightarrow \infty} \frac{F^n(x)}{n}$. E chamamos o número de rotação de f por $\rho(f) = \rho(F)(\text{mod } 1)$.

Definição 6 Seja $f : S^1 \rightarrow S^1$, $J \subset S^1$. Dizemos que J é um intervalo errante se:

1. $J, f(J), f^2(J), \dots$ são disjuntos dois a dois.
2. $\omega(J) = \bigcup_{x \in J} \omega(x)$ não é uma única órbita periódica.

Teorema 1 (Primeiro Teorema de Denjoy [1]): Seja $f : S^1 \rightarrow S^1$ um difeomorfismo de classe C^2 , com $\rho(f) = \alpha \notin \mathbb{Q}$. Então f é conjugado a R_α .

Teorema 2 (Segundo Teorema de Denjoy [1]): Seja $\alpha \notin \mathbb{Q}$. Então existe $f : S^1 \rightarrow S^1$ um difeomorfismo de classe C^1 com $\rho(f) = \alpha$ tal que f tem um intervalo errante, isto é, f não é conjugado a R_α .

Por fim, concluiremos com a demonstração do segundo teorema de Denjoy que é o foco principal do trabalho.

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Bifurcações Genéricas e Relações de Equivalência em Campos de Vetores Suaves por Partes

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Abstract

Neste trabalho abordaremos aspectos qualitativos e geométricos de Campos de Vetores Suaves por Partes (CVSP's ou Campos de Filippov). Consideraremos $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ uma função suave que tem $0 \in \mathbb{R}$ como valor regular. Definimos $\Sigma = f^{-1}(0)$, de tal forma que dividiremos o plano \mathbb{R}^2 em duas regiões: $\Sigma^+ = \{p \in \mathbb{R}^2 / f(p) > 0\}$ e $\Sigma^- = \{p \in \mathbb{R}^2 / f(p) < 0\}$. Nas regiões Σ^+ e Σ^- atuarão os campos contínuos X e Y respectivamente, ambos de classe C^r . Assim, um Campo de Filippov $Z : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ é definido por

$$Z(p) = \begin{cases} X(p), & \text{se } p \in \Sigma^+, \\ Y(p), & \text{se } p \in \Sigma^-. \end{cases}$$

Serão introduzidos os conceitos de equivalência topológica e Σ -equivalência entre Campos de Filippov, bem como os conceitos de estabilidade e Σ -estabilidade estrutural. A partir disso, iremos determinar as formas normais de CVSP's definidos numa vizinhança de pontos regulares ou de singularidades genéricas.

Por fim, classificaremos e caracterizaremos bifurcações genéricas locais e globais de codimensão 1 por meio do retrato de fase e do diagrama de bifurcação dos campos envolvidos, que é o objetivo principal deste trabalho. Este estudo será feito tanto desdobrando CVSP's cuja representação envolve apenas um parâmetro, quanto para CVSP's cuja forma normal apresenta dois parâmetros. Em geral, nos CVSP's estudados aparecerão pontos de equilíbrio sobre Σ ou tangências das trajetórias dos campos com Σ ou até mesmo a simultaneidade destes fenômenos.

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Understanding the behaviour of “good” dynamical systems by discretizing them

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Abstract

In many dynamical systems the transfer operators exhibits regularizing properties on adapted Banach spaces. In one dimensional uniformly hyperbolic systems, one of the ways we can think about these regularizing properties is the fact that the “fine” behaviour of the system can be understood by looking at a “coarse” representation of the system. This is the principle that lies behind some approximation techniques for invariant measures and the rigorous numerical study of some of the properties of the system. In this poster I will present some examples of this principle and some lines of research that I’m currently working on.

Generic Properties of Magnetic Flows

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Abstract

The perturbative theory on dynamical systems is one of the most powerful ones to describe robust and generic properties of dynamical systems. Previous perturbations theorems were Kupka-Smale's theorem and the Franks' lemma. Those perturbations tools were generalized to more difficult settings: vector fields, Hamiltonians, etc. However, there are very important dynamics coming from the differential geometry: geodesic flows. The understanding of this flow quickly by Hopf and Anosov, that give us tools for the understanding of the dynamics and statistics of that flow for negatively curved Riemannian manifolds.

The first ones to deal with Kupka-Smale's theorem for geodesic flows were Klingenberg, Takens and Anosov. They were delicate computations and do not were suitable to obtain a Franks-type result. It was Contreras and Paternain, the first ones to obtain Franks' lemma for geodesic flows, but for surfaces. After many years, Contreras was able to obtain Franks' lemma for any dimension (geodesic flows).

The geodesic equation can be view like the Hamiltonian flow of the Kinetic Energy. Another well known flows, induced from some physical phenomena, are the magnetic flows can be written as the following equation,

$$\begin{cases} \dot{x} = v, \\ \nabla_v v = Y_x(v), \end{cases}$$

where Y is the so called Lorentz force. The magnetic flow was studied by Miranda but only in dimension two [2, 3].

Our main results is to give a full proof of Kupka-Smale's theorem and Franks' lemma for magnetic flows in any dimension.

Our method differs to the previous ones, since we use new results from the Control Theory that was used by Rifford and Ruggiero [4], to obtain similar results for Hamiltonians perturbations.

Let us explain the differences. There are three objects in the analysis: the Hamiltonian (that is the kinetic force in the geodesic and magnetic flow), the symplectic form, and the metric. For geodesic flows, the metric is perturbed. In Rifford and Ruggiero article [4, 1], the Hamiltonian is perturbed. In our work the symplectic form is perturbed. To obtain these results, one of the key arguments is to obtain nice coordinates to transform our analysis in a equation to apply the control theory.

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Famílias Anosov: Estabilidade e Entropía

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Abstract

As famílias Anosov foram introduzidas por P. Arnoux e A. Fisher em [1], motivados por generalizar a noção de difeomorfismos Anosov. Neste poster mostraremos que o conjunto formado pelas famílias Anosov é estável. A seguir, falaremos um pouco sobre estas noções:

Sejam M_i variedades Riemannianas com métricas Riemannianas $\langle \cdot, \cdot \rangle_i$ fixadas para $i \in \mathbb{Z}$. Considere a *união disjunta*

$$\mathbf{M} = \coprod_{i \in \mathbb{Z}} M_i = \bigcup_{i \in \mathbb{Z}} M_i \times i.$$

Denotaremos por $\|\cdot\|_i$ a norma induzida por $\langle \cdot, \cdot \rangle_i$ em TM_i e tomaremos $\|\cdot\|$ definida em \mathbf{M} , como sendo $\|\cdot\|_{M_i} = \|\cdot\|_i$ para $i \in \mathbb{Z}$. Uma *família de homeomorfismos* $(\mathbf{M}, \langle \cdot, \cdot \rangle, \mathbf{f})$, é uma aplicação $\mathbf{f} : \mathbf{M} \rightarrow \mathbf{M}$, tal que, para cada $i \in \mathbb{Z}$, $\mathbf{f}|_{M_i} = f_i : M_i \rightarrow M_{i+1}$ é um homeomorfismo. A n -ésima composição é definida em cada M_i como sendo $\mathbf{f}_i^n = f_{i+n-1} \circ \dots \circ f_i : M_i \rightarrow M_{i+n}$ se $n > 0$, $\mathbf{f}_{i-n}^{-1} \circ \dots \circ \mathbf{f}_{i-1}^{-1} : M_i \rightarrow M_{i-n}$ se $n < 0$ e $Id : M_i \rightarrow M_i$ se $n = 0$, para cada $i \in \mathbb{Z}$.

Uma *família Anosov* é uma família de homeomorfismos $(\mathbf{M}, \langle \cdot, \cdot \rangle, \mathbf{f})$ tal que:

1. As aplicações $f_i : M_i \rightarrow M_{i+1}$ são difeomorfismos de classe C^1 ;
2. o fibrado tangente $T\mathbf{M}$ tem uma decomposição contínua $E^s \oplus E^u$ a qual é $D\mathbf{f}$ -invariante, isto é, para cada $p \in \mathbf{M}$, $T_p\mathbf{M} = E_p^s \oplus E_p^u$ com $D_p\mathbf{f}(E_p^s) = E_{\mathbf{f}(p)}^s$ e $D_p\mathbf{f}(E_p^u) = E_{\mathbf{f}(p)}^u$, onde $T_p\mathbf{M}$ é o espaço tangente no ponto p ;
3. existem constantes $\lambda \in (0, 1)$ e $c > 0$ tais que para cada $n \geq 1$, para cada i , para todo ponto $p \in M_i$, temos: $\|D_p(\mathbf{f}_i^{-n})(v)\| \leq c\lambda^n\|v\|$ para cada vetor $v \in E_p^u$, e $\|D_p(\mathbf{f}_i^n)(v)\| \leq c\lambda^n\|v\|$ para cada vetor $v \in E_p^s$.

Consideremos agora $F(\mathbf{M})$ o conjunto formado pelas famílias de difeomorfismos definidas em \mathbf{M} . Muniremos $F(\mathbf{M})$ com a topología Whitney. Provaremos que o conjunto das famílias Anosov, que denotamos por $An(\mathbf{M})$, é um subconjunto aberto de $F(\mathbf{M})$. Também mostraremos algumas propriedades destas famílias e varios exemplos interessantes. Além disso, vamos introduzir uma fórmula de entropía para famílias de homeomorfismos.

Este resultado faz parte do meu trabalho de tese de doutorado em matemática no IME-USP.

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On the Centralizers of Flows

Davi Obata

11-Jul-2016

Let M be a Riemannian manifold and $X \in \mathcal{X}^1(M)$ be a C^1 -vector field. The set $\mathcal{Z}^1(X)$ is the set of all C^1 vector fields Y that commutes with X , that is $[X, Y] = 0$. One could look at the centralizers as certain symmetries of the system. In this work we study two types of problems regarding centralizers. The first is how some chaotic behaviour, such as several types of expansivity, implies small centralizers, in other words few symmetries, and the second is how "large" centralizers implies lack of "chaos". This is a joint work with Bruno Santiago and Javier Correa.

Sequential Gibbs Sequences and Factor Maps

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Abstract

We define the of *sequential Gibbs measures*, inspired on the classical notion of Gibbs measures and recent examples from the study of non-uniform hyperbolic dynamics. Extending previous results of Kempton-Pollicott and Ugalde-Chazottes, we show that the images of one block factor maps of a sequential Gibbs measure are also a sequential Gibbs measure, with the same Gibbs sequence. We obtain some estimates on the regularity of the potential of the image measure at almost every point such as Local stretched exponential decay, Local polynomial decay and Local summable variations.

Ciclos Limites em Sistemas Diferenciais Planares Lineares por Partes

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Resumo

Neste trabalho consideramos sistemas diferenciais planares lineares por partes em duas zonas, com o objetivo principal de estudar a existência de ciclos limite. Duas situações específicas foram estudadas com detalhes: o caso em que a variedade de descontinuidade é uma reta e também o caso em que a variedade de descontinuidade tem o formato de uma escada.

Este é um projeto em andamento e ainda iremos considerar outros tipos de sistemas lineares por partes, como por exemplo o caso de três zonas.

Sistemas de computação algébrica, como Maple e Matlab, foram utilizados para produzir exemplos dos resultados obtidos.

Este trabalho foi financiado pelo projeto “Equações Diferenciais Não-Lineares” do PROCAD-Capes.

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Hexagonal String Billiard

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Abstract

Uma das conjecturas mais famosas acerca de de cáusticas em bilhares é devido a G.D. Birkhoff (veja [1]). Basicamente ela afirma que os únicos bilhares convexos e suaves integráveis são elipses. Fetter traz em [2] um bilhar construído por pedaços de elipses, onde ainda não se sabe sobre sua integrabilidade. Usando simulações computacionais do espaço de fase e outros experimentos numéricos, Fetter vê o que parecem ser curvas invariantes e não vê características esperadas de bilhares não integráveis.

Pelos motivos acima, tal bilhar possui peculiaridades que tornam seu estudo interessante, incluindo sua possível (porém improvável) integrabilidade. Assim, o propósito deste pôster é apresentar tal bilhar, similares e suas propriedades básicas, exploradas por Fetter em [2].

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First integrals of the May-Leonard asymmetric system

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Abstract

We investigate the existence of first integrals in the three dimensional May-Leonard asymmetric system. Using the computational algebra systems Mathematica and Singular we first look for families of the May-Leonard asymmetric system admitting invariant surfaces of degree two. Then using these invariant surfaces and invariant planes we construct first integrals of the Darboux type identifying subfamilies of such system admitting one first integral and two independent first integrals.

Generic local behavior of a certain class of piecewise smooth system

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Abstract

In this work, we consider piecewise smooth systems whose discontinuity set is an algebraic variety, more specifically, the set of zeros of the map $f(x, y) = x \cdot y$. Let Ω be the set of all piecewise systems $Z = (X, Y)$ with switching manifold $\Sigma = f^{-1}(0)$, defined by the differential equation

$$\dot{\mathbf{x}} = \frac{1}{2}(X + Y)(\mathbf{x}) + \frac{1}{2}\text{sgn}(f(\mathbf{x}))(X - Y)(\mathbf{x}),$$

where $\mathbf{x} = (x, y)$ and X, Y are planar vector fields. Our goal is to study rigorously the local behavior around the origin and classify the codimension zero and one singularities of $Z \in \Omega$. Concerning to the codimension zero singularities, we construct the homeomorphisms which give the weak equivalences between each piecewise vector field Z and its corresponding normal form, preserving the regions of Σ . We also classify the codimension one singularities, exhibit the versal unfoldings in each case and prove that the set of these vector fields is a codimension one embedding manifold of Ω .

Dynamic of Positively Expansive Measures on Uniform Spaces

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Abstract

With the main goal of a detailed reconstruction of the article [3], we give partial answers to the questions raised in it, we define positively expansive on uniform spaces extending the analogous concept on metric spaces developed in [1] and [2]. We show that positively expansive probability measures on Lindelöf spaces are non-atomic and their corresponding maps eventually aperiodic (and hence aperiodic), also that the stable classes of measurable maps have measure zero with respect to any positively expansive invariant measure. Additionally, any measurable set where a measurable map in a Lindelöf uniform space is Lyapunov stable has measure zero with respect to any positively expansive inner regular measure. We conclude that the set of sinks of any bimeasurable map with canonical coordinates of a Lindelöf space has zero measure with respect to any positively expansive inner regular measure. Finally, we show that every measurable subset of points with converging semi-orbits of a bimeasurable map on a separable uniform space has zero measure with respect to every expansive outer regular measure, which generalizes the results established in [4] and [1].

Keywords: Uniform spaces, expansive maps, expansive measures, τ -expansive measures, Reddy Theorem

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Random walks driven by non-uniformly expanding transformations

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Abstract

Kesten's amenability criterion is a classical theorem from probability theory stating that a group is amenable if and only if the Markov operator associated to the symmetric random walk on the group has spectral radius 1. In this work, we extend this approach to random walks which are driven by a non-uniformly expanding transformation. That is we consider the time evolution of the second coordinate of

$$T : M \times G \rightarrow M \times G, (x, g) \mapsto (f(x), g\gamma(x)),$$

where $f : M \rightarrow M$ is non-uniformly expanding and γ is a map from the manifold M to G and relate amenability with the exponential decay of $\mu(\{x : T^n(x, id) \in M \times \{id\}\})$, with μ referring to a expanding measure for f .

Random walks driven by non-uniformly expanding transformations

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Abstract

Kesten's amenability criterion is a classical theorem from probability theory stating that a group is amenable if and only if the Markov operator associated to the symmetric random walk on the group has spectral radius 1. In this work, we extend this approach to random walks which are driven by a non-uniformly expanding transformation. That is we consider the time evolution of the second coordinate of

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where $f : M \rightarrow M$ is non-uniformly expanding and γ is a map from the manifold M to G and relate amenability with the exponential decay of $\mu(\{x : T^n(x, id) \in M \times \{id\}\})$, with μ referring to a expanding measure for f .

Asymptotic expansion of the heteroclinic bifurcation for the planar normal form of the 1:2 resonance

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Abstract

We consider the family of planar differential systems depending on two real parameters

$$\dot{x} = y, \quad \dot{y} = \delta_1 x + \delta_2 y + x^3 - x^2 y.$$

This system corresponds to the normal form for the 1:2 resonance which exhibits a heteroclinic connection. The phase portrait of the system has a limit cycle which disappears in the heteroclinic connection for the parameter values on the curve $\delta_2 = c(\delta_1) = -\frac{1}{5}\delta_1 + O(\delta_1^2)$, $\delta_1 < 0$. We significantly improve the knowledge of this curve in a neighborhood of the origin.

Ciclos limites em sistemas lineares por partes com duas zonas no plano

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Abstract

Neste trabalho iremos tratar da unicidade de ciclos limites para sistemas lineares por partes com duas zonas no plano, em que a curva de separação é uma reta formada por arcos de costura e com uma única Σ -singularidade monodrómica, o qual foi introduzido por Medrado e Torregrosa em [1]. Tal assunto vem sendo tratado no passado recente por vários pesquisadores. Lum e Chua, em [2], conjecturaram que sistemas lineares por partes contínuos com duas zonas tem no máximo um ciclo limite, o qual foi provado em [3] por Freire, Ponce, Rodrigo e Torres.

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Transcritical and zero-Hopf bifurcations in the Genesio system

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Abstract

In this work we study the existence of transcritical and zero–Hopf bifurcations of the third–order ordinary differential equation

$$x''' + ax'' + bx' + cx - x^2 = 0,$$

called the Genesio equation, which has a unique quadratic nonlinear term and three real parameters. More precisely, writing this differential equation as a first order differential system in \mathbb{R}^3 we prove: first that the system exhibits a transcritical bifurcation at the equilibrium point located at the origin of coordinates when $c = 0$ and the parameters (a, b) are in the set $\{(a, b) \in \mathbb{R}^2 : b \neq 0\} \setminus \{(0, b) \in \mathbb{R}^2 : b > 0\}$, and second that the system has a zero–Hopf bifurcation producing a periodic orbit near the equilibrium point located at the origin when $a = c = 0$ and $b > 0$.

Pesin's Entropy Formula of C^1 -expanding maps.

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Abstract

There are extensive results concerning Pesin's entropy formula, but the vast majority of these results were obtained out under the assumption that the dynamics is of regularity $C^{1+\alpha}$. It is interesting to investigate Pesin's entropy formula under the weaker C^1 differentiability hypothesis. In fact, still there is a gap between the dynamics $C^{1+\alpha}$ and the C^1 , despite recent progress in this direction.

In this work, we extend the result (one-dimensional) obtained by Catsigeras E. and Heber E., showing that for any C^1 -expanding map $f : M \rightarrow M$ on some d -dimensional ($d \geq 1$) compact Riemannian manifold M all the (necessarily existing) SRB-like measures satisfies Pesin's Entropy Formula, i.e.,

$$h_\mu(f) = \int \log |\det Df| d\mu.$$

for all $\mu \in \mathcal{M}_f$ SRB-like measures.

Ciclos limites em sistemas diferenciais polinomiais de Liénard descontínuos.

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Neste trabalho nos dedicamos aos estudos de ciclos limites de um sistema polinomial de Liénard de grau n com m zonas, para $m = 2, 4$, dado por

$$\begin{cases} \dot{x} = y + \operatorname{sgn}(g_m(x, y))F(x) \\ \dot{y} = -x \end{cases}, \quad (1)$$

onde $F(x) = a_0 + a_1x + \dots + a_nx^n$, $a_n \neq 0$ e o conjunto de zeros da função $\operatorname{sgn}(g_m(x, y))$ com $m = 2, 4$, é a união de $\frac{m}{2}$ diferentes retas passando pela origem dividindo o plano em setores angulares de tamanho $\frac{2\pi}{m}$ e $\operatorname{sgn}(z)$ denota a função sinal.

A principal ferramenta utilizada é a combinação da Teoria da Média de primeira ordem com o processo de regularização de sistemas descontínuos, considerando as condições de Filippov. Analisaremos um caso particular de um sistema polinomial de Liénard de grau 3 com 4 zonas. Este trabalho foi desenvolvido segundo a referência [2].

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Área de Pesquisa: Sistemas Dinâmicos

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Ciclos limites em sistemas diferenciais polinomiais de Liénard descontínuos.

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Dinâmica do Círculo: O Teorema de Denjoy

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Palavras Chave: Sistemas Dinâmicos, Dinâmica do Círculo, Teorema de Denjoy.

Introdução

Este trabalho visa o estudo do Teorema de Denjoy, mas para isso apresentaremos alguns conceitos importantes da dinâmica topológica e da dinâmica do círculo que serão necessários para o estudo desse Teorema.

Material e Métodos

O material utilizado foi o artigo “Tópicos de Sistemas Dinâmicos” de Martin Sambarino e a metodologia foram a de estudos individuais e apresentação de seminários semanais para a pesquisadora e orientadora.

Resultados e Discussão

Inicialmente, abordaremos algumas definições importantes para o estudo da dinâmica topológica.

Definição 1: Seja M um espaço topológico (de Hausdorff, completo) dizemos que $F: \mathbb{Z} \times M \rightarrow M$, tal que F é contínua, é um sistema dinâmico discreto se:

1. $F(0,.) = id$
2. $F(n, F(m, x)) = F(n + m, x), \forall n, m \in \mathbb{Z}, \forall x \in M.$

Definição 2: Seja $f: M \rightarrow M$ um sistema dinâmico discreto e $x \in M$, definimos o ω -limite de x como:
 $\omega(x, f) = \{y \in M: \exists n_k \rightarrow +\infty; f^{n_k}(x) \rightarrow y\}$.

Dinâmica de uma Rotação: $R_\alpha(x) = x + \alpha \text{ (mod 1)}$.

Agora definiremos conceitos para o estudo da dinâmica do círculo.

Definição 3: Seja f um homeomorfismo do S^1 , dizemos que f é conjugado à rotação de ângulo α se existir um homeomorfismo $h: S^1 \rightarrow S^1$ tal que $f = h \circ R_\alpha \circ h^{-1}$. Ou, equivalentemente, $f \circ h = h \circ R_\alpha$.

Definição 4: Seja $F: \mathbb{R} \rightarrow \mathbb{R}$ contínua que verifica $\pi \circ F = f \circ \pi$, então dizemos que F é levantamento de f .

Definição 5: Seja F um levantamento de $f \in Hom_+(S^1)$. Definimos um número de rotação de

um levantamento por F , como $\rho(F) = \lim_{n \rightarrow +\infty} \frac{F^n(x)}{n}$. E chamamos o número de rotação de f por $\rho(f) = \rho(F) \text{ (mod 1)}$, onde F é um levantamento de f .

Definição 6: Seja $f: S^1 \rightarrow S^1, J \subset S^1$. Dizemos que J é um intervalo errante se:

1. $J, f(J), f^2(J), \dots$ são disjuntos dois a dois.
2. $\omega(J) = \bigcup_{x \in J} \omega(x)$ não é uma única órbita periódica.

Teorema 1 (Denjoy, [D]): Seja $f: S^1 \rightarrow S^1$ difeomorfismo de classe C^2 , com $\rho(f) = \alpha \notin \mathbb{Q}$. Então f é conjugado a R_α .

Teorema 2 (Denjoy, [D]): Seja $\alpha \notin \mathbb{Q}$. Então existe $f: S^1 \rightarrow S^1$ difeomorfismo de classe C^1 com $\rho(f) = \alpha$ e f tem um intervalo errante (isto é, f não é conjugado a R_α).

Conclusão

Neste trabalho apresentamos resultados conclusivos sobre a dinâmica do círculo e em específico o Teorema de Denjoy.

Agradecimentos

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Dominated splitting for exterior powers and singular hyperbolicity

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Abstract

In this work, jointly to V. Araujo, we show that an invariant splitting for the tangent map to a smooth flow over a compact invariant subset is dominated if, and only if, the exterior power of the tangent map admits an invariant dominated splitting. For a C^1 vector field X on a 3-manifold, we obtain singular hyperbolicity using only the tangent map DX of X and a family of indefinite and non-degenerate quadratic forms without using the associated flow derivative DX_t . As a consequence, we show the existence of adapted metrics for singular-hyperbolic sets for three-dimensional C^1 vector fields.

Uma desigualdade de Ancona para operadores

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Abstract

Neste trabalho, estudamos processos estocásticos em grupos hiperbólicos por meio de teoria de potencial. Um grupo enumerável G é Gromov-hiperbólico se existe δ tal que qualquer triângulo geodésico no grafo de Cayley é δ -fino, isto é, cada lado do triângulo está contido na união da δ -vizinhança dos outros dois lados. Estudamos a evolução do tempo do processo estocástico em um tal G definido por um skew product $T : X \times G, X \times G, (x, g) \mapsto (\theta x, g\gamma(x))$, para $\gamma : X \rightarrow G$. Neste contexto, nosso objetivo é obter um teorema local do limite por meio de bons comportamentos assintóticos do operador de Ruelle \mathcal{L} , que tem o papel do operador do Markov da teoria da probabilidade. Por este fim, definiremos uma família de operadores de Green da seguinte forma, para $1 \leq r < R$,

$$\mathbf{G}_r : \mathcal{H} \rightarrow \mathcal{H}, f \mapsto \mathbf{G}_r(f) := \sum_{n=0}^{\infty} r^n \mathcal{L}^n(f),$$

onde R é o raio de convergência da série e \mathcal{H} é um espaço de funções Hölder contínuas. Provaremos que todo grupo Gromov-hiperbólico G satisfaz uma desigualdade de Ancona uniforme, isto é, existe uma constante $C > 0$ tal que, para qualquer $g_1, g_2 \in G$ e para qualquer h perto de um segmento geodésico de g_1 para g_2 e $r \in [1, R]$,

$$\mathbf{G}_r(\mathbf{1}_{X \times \{g_1\}})(x, g_2) \leq C \mathbf{G}_r(\mathbf{1}_{X \times \{g_1\}})(x, h) \mathbf{G}_r(\mathbf{1}_{X \times \{g_1\}})(x, g_2).$$

Este resultado é o primeiro passo para obter um teorema de limite local, isto é, o comportamento assintótico de $\mathcal{L}^n(\mathbf{1}_{X \times \{g\}})(x, h)$.