Automatic smoothing by optimal splines

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ABSTRACT

We propose a method that is capable to filter out noise as well as suppress outliers of sampled real functions under fairly general conditions. From an a priori selection of the number of knots that define the adjusting spline, but not their location in that curve, the method automatically determines the adjusting cubic spline in a least-squares optimal sense. The method is fast and easily allows for selection of various possible number of knots, adding a desirable flexibility to the procedure. As an illustration, we apply the method to some typical situations found in geophysical problems.

INTRODUCTION

In experimental sciences we are often required to represent a set of measured data in the form of a smooth curve from which desirable parameters or attributes are to be extracted. A common problem is the presence of noise in the data. In the literature we find several methods that try to filter and/or smooth the data. Many of them provide us, after application, a different sampled dataset that, following some criteria, can be seen as smoother than the original one. We can also think of an interpolation approach, where the noisy data is replaced by corresponding points that belong to an interpolating function.

The natural question is how should we choose the "interpolating points" from the data so as to construct the desired smoothing function. Normally, these points are extracted from the data, in a regular fashion or manually selected.

We propose a method that optimally selects points to define a cubic spline that best represents the data in the least-squares sense. An interesting feature of the method is that the knot points are no longer required to belong to the original data set. As we will see below, the method is suitable, not only for smoothing, but also for discarding outliers. The method is designed to handle datasets composed by samples of rather complicated real functions. It is to be stressed that, by construction, the obtained function is naturally smooth up to second-order derivative.

The proposed method is applied to two important problems in geophysics. The first problem is to recover horizons as part of a macro-velocity model inversion from multi-coverage seismic data and to smooth seismic traveltime attributes (see **?**). The second application refers to smoothing well-log data for anomaly detection and inversion purposes.

FORMULATION

Consider a noisy data $\Omega = \{(x_j, y_j) \in I\!R^2 \,|\, j = 1, \ldots, M\}$. Let N be the number of interpolating points and $\Gamma = \{(X_i, Y_i) \in I\!R^2 \,|\, X_{i-1} < X_i,\, i = 1, \ldots, N\}$ be the set of these points that defines the sought-for cubic spline. To obtain the best set Γ, in the least-squares sense, we must solve the $2N$-variable problem

$$\min_{\Gamma} \sum_{j=1}^{M} |y_j - s(x_j)|^2 \,, \qquad \text{s. t.} \quad \begin{cases} s \text{ is the cubic spline defined by } \Gamma \\ X_1 \geq \min_j x_j \\ X_N \leq \max_j x_j \end{cases} . \tag{1}$$

To solve this problem, we have employed the optimization solver called GENCAN proposed by **?**. GENCAN is an active-set method for smooth box-constrained minimization. The algorithm combines an unconstrained method, including a line search which aims to add many constraints to the working set at a single iteration, with a recently introduced technique (spectral projected gradient) for dropping constraints from the working set. As usual, the optimization process needs an initial approximation. For this purpose, we chose the initial set as composed by N regularly sampled pairs on the originally given set Ω, that is, $(X_i, Y_i) = (x_j, y_j)$, with $j = \lfloor 1 + (i-1) \cdot (M-1)/(N-1) \rfloor$, for $i = 1, \ldots, N$, where $\lfloor x \rfloor$ denotes the greater integer less than or equal to x.

Note that we have not made any consideration on how to choose the number, N, of interpolating points. The method is designed to automatically find, in the least-squares sense, the best cubic spline for the specified number of knots N. Of course, for a small number of points N, the obtained spline will not be able to represent more than the general trend of the curve. On the other extreme, for large values of N, the spline will tend to fit even the outliers. Since the method is fast, it is reasonable to estimate the cubic spline for several choices of N. This flexibility can be very useful to the user or interpreter, in the sense that a number of inexpensive trials can be implemented before a final decision on which level of smoothness is the best choice for the problem.

APPLICATIONS

We now illustrate the application of the proposed method to some common practical situations. We start by testing the ability of the method to smooth a sequence of the four datasets of increasing difficulty, shown in Figure 1. We next apply the procedure to two problems related to seismic imaging and inversion, shown if Figures 2 and 3, respectively.

General situations

Figure 1(a) shows that the method efficiently handles and removes white noise. In the next example (Figure 1(b)), besides the noise, some outliers were added. Again, the optimized cubic spline represents the data very well. In Figure 1(c), we see that the obtained curve is able to well describe abrupt variations of the data. Finally, in Figure 1(d), we see that the method is robust enough to provide good results even in the presence of discontinuities. Note that, in particular, there are no Runge effects near the discontinuities.

Horizon reconstruction

We present the results of an algorithm for reconstruction of interfaces and inversion of attributes from 2D-multi-coverage seismic data. As reported in **?**, the procedure has been successfully applied to invert a layered macro-velocity model from the data. Figure 2(a) depicts the inverted model, where the estimated interfaces (solid lines) were approximated by *conventional* cubic splines (constructed by selection of knots among the sampled set). In Figure 2(b), we can see the improvement of the inverted model, when the interfaces where constructed upon the application of the proposed technique.

Well-log analysis

Well logs play the important role of linking rock parameters to seismic data. As an example, impedance functions derived from well logs are generally used for identification and characterization of reservoir anomalies. Well data (e.g., P-and S-velocities and density) are, in general, very noisy, so it may be desirable to consider the parameters as smooth functions of depth (or time). Figure 3 shows the application of the method to smooth a couple real data well logs. On the top of the figure, the new method was used with $N = 20$ knots. On the bottom of the figure, the number of knots was $N = 30$.

CONCLUSIONS

We presented an automatic method for smoothing and outlier suppression of data sets that consist of sampled real function points. After an *a priori* selection of the number of knots, the procedure automatically finds the location of the interpolating points, in such a way that the resulting smoothing function (a cubic

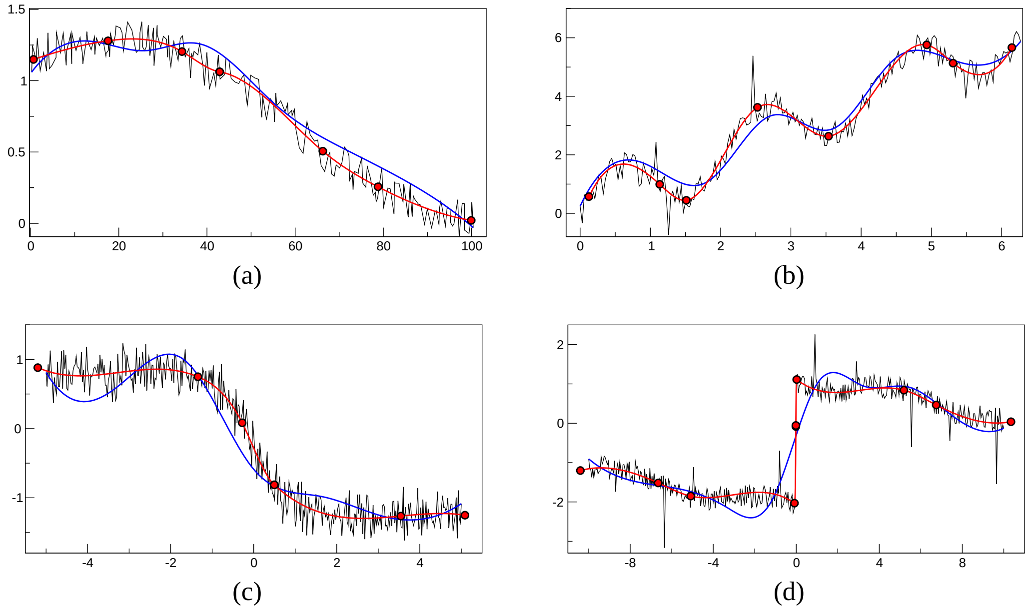


Figure 1: Examples of cubic spline optimal adjust. In all graphs the black line represent the noisy data (linearly interpolated), the blue line states for the initial approximation for the optimization solver, the red line is the optimized cubic spline obtained, and the red dots are the knots that define that spline.

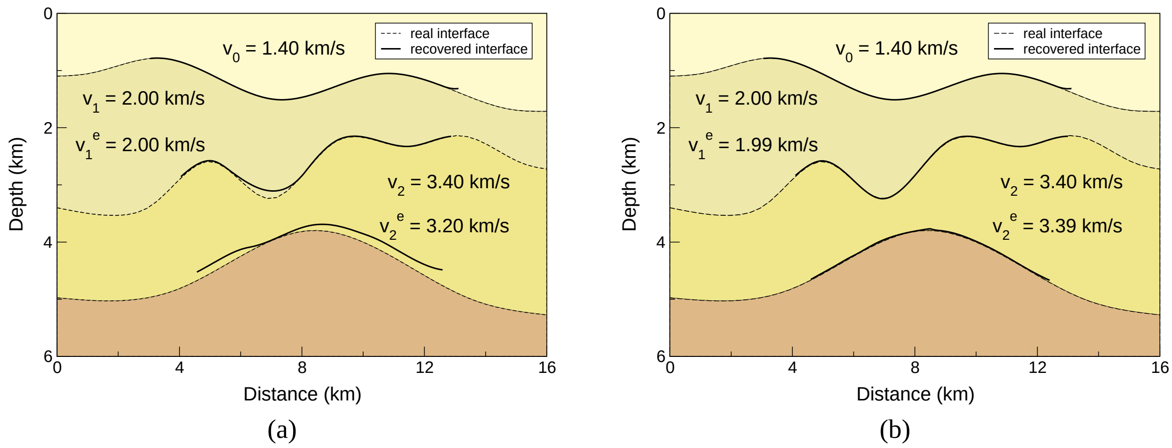
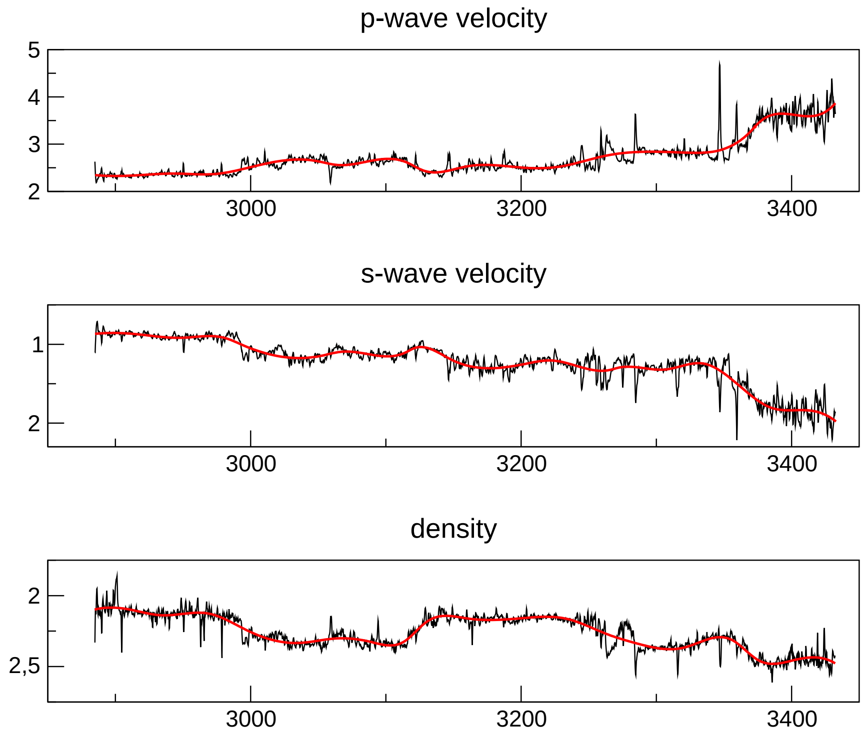
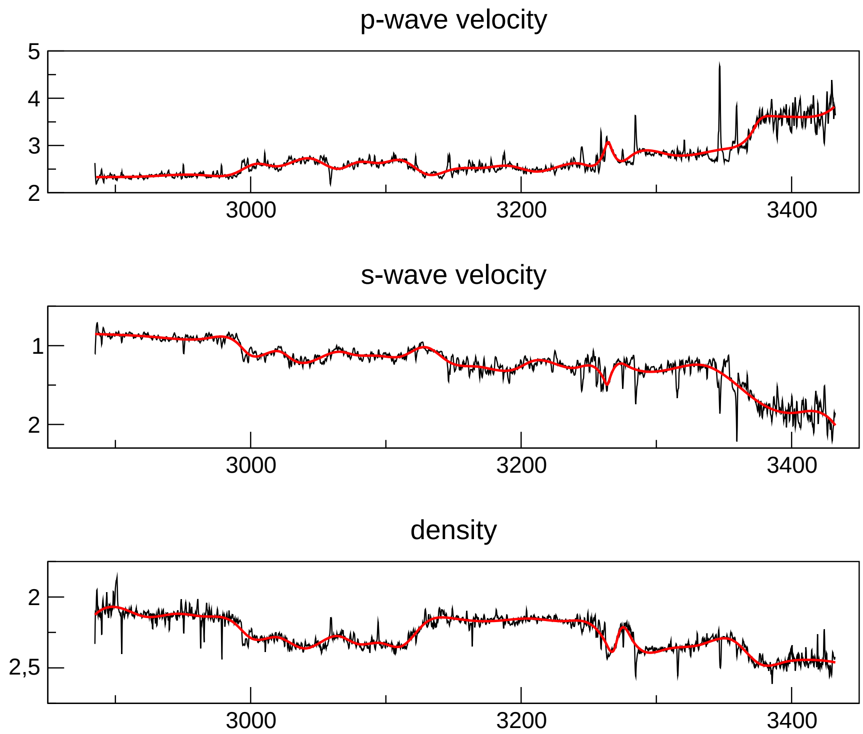


Figure 2: Multiparametric traveltime attributes inversion. The dashed lines represent the real interfaces and the solid lines represent the obtained ones. In (a) the interfaces were approximated by interpolating splines, and in (b) they were approximated by optimal splines. The real and the estimated velocities are v_i and v_i^e, respectively.



(a)



(b)

Figure 3: P- and S-velocities and density well logs, in black, and ajusted cubic spline, in red, with (a) $N = 20$ and (b) $N = 30$.

spline) is optimal in the least-squares sense. As the method is fast, it allows the user to apply the procedure to different numbers of knots, so as to choose the degree of smoothness that best fits the data. A particular feature of the method is its ability to adjust to abrupt discontinuities on the data.

The few illustrations presented in the text show a wide applicability of the method. As further applications, the method could be useful for purposes such as tomographic inversion and automatic picking.

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