THE CAMPBELL-BAKER-HAUSDORFF FORMULA FOR RELATIVELY FREE LIE ALGEBRAS

ŞEHMUS FINDIK

Consider the algebra $K\langle\langle x, y\rangle\rangle$ of formal power series in two noncommuting variables x, y, over a field K of characteristic zero. The Hausdorff series $H(x, y)$ is the solution to the equation $z = \log(e^x e^y)$, which is given by the Campbell-Baker-Hausdorff formula

$$H(x,y) = x + y + \frac{1}{2}[x,y] + \frac{1}{12}([x,[x,y]] + [y,[y,x]]) + \frac{1}{24}[y,[x,[y,x]]] + \cdots$$

This series belongs to the completion $\hat{L} \subset K\langle\langle x, y\rangle\rangle$ of the free Lie algebra L of rank 2 generated by x, y. We evaluate $H(x, y)$ on various relatively free Lie algebras in order to obtain closed-form expressions in the variables x, y.

This is a joint work with Vesselin Drensky* and Lothar Gerritzen**.

Department of Mathematics, Çukurova University, 01330 Balcali, Adana, Turkey
E-mail address: sfindik@cu.edu.tr

*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria
E-mail address: drensky@math.bas.bg

**Fakultät für Mathematik, Ruhr-Universität, 44780 Bochum, Germany
E-mail address: lothar.gerritzen@rub.de