VIRTUAL ENDOMORPHISMS OF GROUPS

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A virtual endomorphism of a group G is a homomorphism $f : H \to G$ where H is a subgroup of G of finite index m.

A recursive construction using f produces a so called state-closed (or, self-similar in dynamical terms) representation of G on

a 1-rooted regular m-ary tree. The kernel of this representation is the f-core (H); i.e., the maximal subgroup K of H which is

(i) normal in G and (ii) f-invariant, in the sense that $K^f \leq K$.

Examples of state-closed groups are the Grigorchuk 2-group and the Gupta-Sidki p-groups in their natural representations on rooted trees.

The affine group $\mathbb{Z}^n GL(n, \mathbb{Z})$ as well as the free group F_3 in three generators admit state-closed representations.

Yet another example is the free nilpotent group $G = F(c; d)$ of class c.

We will discuss state-closed representations of

(1) *general abelian groups*, (2) *finitely generated nilpotent groups*, (3) *wreath product of abelian groups*.

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